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A New Interregional Input-Output Model with Endogenous Self-sufficiency Rate

Qi Su, Jian Xu, Jialu Sun, Chuan Li

School of Economics and Management

University of Chinese Academy of Sciences

E-mail:suqi22@mails.ucas.ac.cn

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Abstract： **C**lassic input-output models often assume that domestic and imported products are either perfectly substitutable or non-substitutable in measuring the effects of external demand shocks. However, these assumptions do not fully reflect real-world trade patterns. In this paper, we assume that domestic and imported products of the same sector are differentiated products therefore whose elasticity of substitution is non-zero. Based on this assumption, we develop a new interregional input-output model that allows for a constant elasticity of substitution between domestic and imported products by supposing each industry has a Cobb-Douglas production function and intermediate inputs of each industry has a Constant Elasticity of Substitution (CES) aggregator over domestic and imported products. One attractive aspect of our model is that the self-sufficiency rate of intermediate inputs is endogenized by profit maximization of all firms which is determined by both substitution elasticity between domestic and imported products and the international price indices. The new model could provide a more flexible framework to analyze the effects of external demand shocks.

To empirically check the differences in the results of measuring the effects of external demand shocks between our model and the classic Interregional Input-output model, we focus on demand-side shocks which refer to changes in exports from representative industries the between China and the U.S. Operating the new model need to assign values to three types of parameters for: the input coefficient matrix, the relative price level among countries and the elasticity of substitution between countries of each sector. We use the 2014 World Input-Output Database (WIOD) to construct a three-region, 56-sector IRIO table covering China, the United States, and the rest of the world (ROW) to get the input coefficient matrix required. The information of the relative price level among countries comes from the cross-country price indices of the United Nations International Comparison Program((ICP) and the distribution range of the elasticity of substitution between domestic and imported products from the existing empirical results of the Armington elasticity.

The results indicate that the shocks estimated by the new model have a lower impact on China's value-added compared to the classic model but a higher impact on the U.S.' value-added than the classic estimate. At the sectoral level, the two models

identify significantly different sectors as the most affected by trade shocks. To observe the impact of changes in the elasticity of substitution on external demand shocks, we add two scenarios for simulation: a) a high substitutability relationship between domestic and imported products; b) a low substitutability relationship between domestic and imported products. By comparing the simulation results, we find that changes in the elasticity of substitution affect the estimation of the shock's impact, but the classic model consistently provides higher estimates of the impact on China's value-added and lower estimates of the impact on the U.S.' value-added compared to the new model.
Key words: Interregional input-output model; Armington Elasticity; Relative Price Level among Countries; External demand Shock； Self-sufficiency Rate

1. Introduction

How and to what extent exogenous shocks are propagated through the economy has attracted wide attention and has been the hot topic of research related to policy-making. The deepening of globalization in the 21st century has turned multinational trade networks into the nervous system of the global economy. What are the consequences of even small shocks from other country's exports demand? According to the data from the World Trade Organization (WTO, 2023), the global trade value of goods reached $25.3 trillion in 2022, while the trade value of services surpassed $7 trillion, accounting for 28% of global GDP. This complex system is undergoing unprecedented stress testing: the Russia-Ukraine conflict caused Europe’s energy import price index to surge by 412% year-on-year in March 2022 (Eurostat, 2023); the U.S. CHIPS and Science Act has triggered a restructuring of the semiconductor supply chain, increasing the monthly export volatility of Asia's electronics industry by 22 percentage points (J.P. Morgan, 2023); and the shipping disruptions caused by the COVID-19 pandemic extended the global average delivery cycle from 40 days to 73 days (World Bank, 2022). These shocks not only caused direct economic losses but also generated cascading effects through global value chains (GVCs). According to the International Monetary Fund (IMF, 2023), supply chain disruptions from 2020 to 2022 permanently reduced the global potential output growth rate by 0.8%.

Scholars develop various methods to measure the effect of external demand shocks. A class of promising approaches based on historical data uses quasi-natural experiments to identify causal effects of trade shocks. Autor et al. (2013), in their landmark study, utilized data on Chinese exports to the U.S. from 1990 to 2007 and employed a region-industry difference-in-differences (DID) model, finding that Chinese import competition led to a 1.5-2.5 percentage point decline in U.S. manufacturing employment. This conclusion has been widely cited in subsequent studies. Such methods face significant endogeneity challenges. Borusyak et al. (2022) pointed out that the traditional construction of shock variables might lead to estimation bias due to omitted variables, and their proposed "shock intensity-exposure" interaction design can reduce bias by 40%. Recently developed machine learning methods offer new approaches for handling high-dimensional data. For instance, Faber et al. (2023) used a random forest algorithm to identify nonlinear transmission paths of tariff changes, finding that the tariff elasticity of intermediate products is 1.8 times higher than that of final goods. These methods provide the useful evaluation instruments, but they heavily rely on the size and the quality of data. Constrained by the available data, econometric models may only make inferences about aggregate variables and have serious limitations in analyzing structural impacts of shocks and in pursing complicated conduction paths.

From the perspective of general equilibrium or industrial linkages, Computable General Equilibrium(CGE) and Interregional Input-output analysis(IRIOA) are commonly used tools. CGE models assess the long-term equilibrium effects of policy shocks by depicting the production and consumption links between multiple countries and sectors. The Global Trade Analysis Project model (Hertel, 1997) is a typical example, using nested CES functions to simulate substitution elasticities between sectors. Corong et al. (2017) used a dynamic GTAP model to predict that the Trans-Pacific Partnership (TPP) would increase the real GDP of member countries by 0.4%-1.1%. However, CGE models have rigid theoretical assumptions, require large and opaque parameter settings, and Kehoe et al. (2018) demonstrated that when price rigidity exceeds model assumptions, welfare losses from tariff shocks may be underestimated by 27%. Furthermore, the quality of benchmark equilibrium data directly impacts the reliability of results. Arndt et al. (2022) found that the agricultural value-added of African countries in the GTAP 11 database is systematically underestimated, leading to simulation errors of 15%-20% in food trade shocks.

Interregional input-output models, due to their structural transparency and network-tracking capabilities, are also valuable tools for studying the transmission of trade shocks. Leontief's (1936) input-output analysis provides the foundational methodology for these models, emphasizing the interdependence between sectors in terms of supply and demand. In a multinational context, demand fluctuations, supply chain disruptions, and other external demand shocks are amplified through the "Leontief inverse matrix" of the IRIO model. This model not only measures the direct effects of exogenous shocks but also quantifies the multiplier effects along the production chain (Miller & Blair, 2009). The World Input-Output Database (WIOD), constructed by Timmer et al. (2015), provides a standardized data framework for multinational studies. For example, changes in a country's exports can be transmitted to upstream and downstream countries via global value chains (GVCs) (Wang et al., 2017). Dietzenbacher et al. (2013) used the MRIO model to quantify the external demand spillover effects of demand contraction during the financial crisis. Bown and Crowley (2020) combined the IRIO model with tariff data to quantify the indirect effects of the reciprocal tariffs between China and the U.S.

Classic input-output models often assume that domestic products and imports are either perfectly substitutable as in the competitive input-output table or non-substitutable as in the non-competitive input-output table. Moreover, in trade theory, the Armington specification which differentiates products by country of origin has been widely accepted in theoretical and empirical analysis, therefore elasticity of substitution between imports and domestic products is usually set to the non-zero parameter. IO modelers often harmonized the obvious contradiction by the following technical routines: incorporating econometric models to estimate the effect of external demand shocks on exogenous variables as export typically where considering Armington specification, then the estimated change in exogenous variables are introduced into IRIO to get the total effect on the target variables by input-output linkages. The main shortcomings of this technical routines lie in that substitution elasticity typically are still assumed to be zero when calculating the diffusion impact of exogenous shocks along production networks in IRIO modeling. This will lead to estimation bias due to which clearly contradicts real-world conditions.

In this paper, we develop a new interregional model by extending the single region input-output network model to multi-region and introducing the Armington specification that permits a non-zero substitution elasticity between imported and domestic products in Leontief inverse, addressing the limitations of classic methodologies. In this model, the self-sufficiency rates are not treated as parameters as the classic IRIO model done but are endogenized by a two-stage profit-maximization mechanism to the function of relative price level among countries and the elasticity of substitution between imported and domestic products. By endogenizing the self-sufficiency rate, our model not only may provide a more effective analysis framework that assures a unanimous theory foundation and therefore obtains a more reliable estimation of the effect of external demand shocks, but also enables a systematic analysis of the impact of substitution elasticities and relative price levels among countries can be explored through the lens of comparative statics. This approach also offers policy-relevant insights into how nations can strategically adjust their self-sufficiency trajectories.

The rest of the paper proceeds as follows. Section II presents a new interregional input-output model with an endogenous self-sufficiency rate. Section III describes the scenario setting for the shocks and parameters estimation for the proposed model. Section IV presents the simulation results of the new model and conducts a comparative analysis with classical model outcomes.The last section presents our conclusion.

2.Stating a New Interregional Input-output Model with Endogenous Self-sufficiency Rate

In this section, we develop a new Interregional Input-Output Model with Endogenous Self-Sufficiency Rate(IRIO-SSR) which falls on relative price level among regions and elasticity of substitution, and discuss the difference between the IRIO-SSR and the classic model. Although the IRIO-SSR is a multi-country model, we focus on developing a two-country model to assure our model in a simple fashion. The two-country model may be generalized to n-country model easily.

2.1 Economic Mechanism

Consider a static perfectly competitive economy with two countries denoted as country r and country s. Each country has n industries. For the same industry in country r and country s, the product is differentiated therefore has different price level denoted as p_i^r and p_i^s.

As Acemoglu et al.(2012， 2016), we suppose that each industry, denoted $j = 1, \dots, n$, follows a Cobb-Douglas production function expressed as:

$$z_j^t = {l_j^t}^{\alpha_{ij}^t} \prod_{i=1}^{n} {z_{ij}^t}^{a_{ij}^t} \qquad t = (r, s) \tag{1}$$

Superscript t stands for either country r or country s. z_j^t is the physical quantity of products produced by sector j in country $t . z_{ij}^t$ is the physical quantity of products produced by industry i used as inputs by industry j in country t. l_j^t is labor input of sector j in country t. We suppress capital to simplify the notation and discussion. However, adding capital does not affect the results of the model (Acemoglu et al.2016).

a_{ij}^{t} and α_{lj}^{t} represent the output elasticity of intermediate inputs and labor input respectively. We assume that, for each i, α_{lj}^{t} >0 , and $a_{ij}^{t} \geq 0$ for all j and

$$\alpha_{lj}^{t} + \sum_{j=1}^{n} a_{ij}^{t} = 1 \qquad t = (r, s) \tag{2}$$

so that the production function of each industry has constant returns to scale.

In a two-country economy, z_{ij}^{t} should be viewed as aggregate inputs made up of domestic and imported inputs. Taking the Armington specification, we consider the product of the same industry in a different country are differentiated products and CES aggregator could be applied to express z_{ij}^{t} as follows:

$$z_{ij}^{r} = \left[z_{ij}^{r,r\frac{\varepsilon_{ir}-1}{\varepsilon_{ir}}} + z_{ij}^{s,r\frac{\varepsilon_{ir}-1}{\varepsilon_{ir}}} \right]^{\frac{\varepsilon_{ir}}{\varepsilon_{ir}-1}} \tag{3}$$

$$z_{ij}^{s} = \left[z_{ij}^{r,s\frac{\varepsilon_{is}-1}{\varepsilon_{is}}} + z_{ij}^{s,s\frac{\varepsilon_{is}-1}{\varepsilon_{is}}} \right]^{\frac{\varepsilon_{is}}{\varepsilon_{is}-1}} \tag{4}$$

Here the first letter of the superscript represents the country which the inputs come from and the second letter of the superscript represents the country which the industry consuming inputs locates in. ε_{ir} and ε_{is} are defined as the substitution elasticity between domestic and imported products of the i^{th} product of country r and country s respectively. This kind of denotation imply that we assume that the substitution elasticity as parameters are product-specific and country-specific, but not depend on which industry consumes the inputs.

Representative firm of every industry has to face two-stage optimization problems to maximize its profit. Firstly, representative firm needs to obtain the optimal solution for aggregate intermediate inputs. Secondly, given the aggregate intermediate inputs, the ratio of domestic and imported products will be decided. This is our key setting about economic behaviors.

In the first stage, the target function is profit of the representative firm which could be written as:

$$\pi^{t} = z_{j}^{t} p_{j}^{t} - w^{t} l_{j}^{t} - \sum_{i=1}^{n} p_{i}^{ct} z_{ij}^{t} \qquad t = (r, s) \tag{5}$$

Substituting Equation (1) into (5), profit function is :

$$\pi^{t} = \left(l_{j}^{t\,\alpha_{lj}^{t}} \prod_{i=1}^{n} z_{ij}^{t\,a_{ij}^{t}} \right) p_{j}^{t} - w^{t} l_{j}^{t} - \sum_{i=1}^{n} p_{i}^{ct} z_{ij}^{t} \qquad t = (r, s) \tag{6}$$

Due to the perfectly competitive assumption, we take all prices as given. Representative firm chooses z_{ij}^t and l_j^t to maximize Equation (6).p_j^t is the price of the output by industry j. p_i^{ct} denotes the price of i^{th} aggregate intermediate inputs which could expressed as follows (Dixit and Stiglitz, 1977) to assuring the condition in Equation (8) is satisfied:

$$p_i^{ct} = ((p_i^r)^{1-\varepsilon_{it}} + (p_i^s)^{1-\varepsilon_{it}})^{\frac{1}{1-\varepsilon_{it}}} \qquad t = (r,s) \qquad (7)$$

$$p_i^{ct} z_{ij}^t = p_i^r z_{ij}^{rt} + p_i^s z_{ij}^{st} \qquad t = (r,s) \qquad (8)$$

The first-order condition implies that:

$$a_{ij}^t = \frac{p_i^{ct} z_{ij}^t}{p_j^t z_j^t} \qquad t = (r,s) \qquad (9)$$

It means that the output elasticity of intermediate inputs is the direct input coefficient defined in the input-output analysis.

At the second stage of profit-maximizing, the constrained optimum problem could be described as: Given the fixed aggregate intermediate inputs $\bar{z}_{ij}^t$ which has been decided at the first stage, representative firm chooses the physical quantities of imported products and domestic products to minimize the total cost happened in i^{th} intermediate input. So the target function could be written as formula (10) and the constraint functions could be written as formula (11).

$$TC_{ij}^t = p_i^r z_{ij}^{rt} + p_i^s z_{ij}^{st} \qquad t = (r,s) \qquad (10)$$

$$\bar{z}_{ij}^t = \left[z_{ij}^{rt\,\frac{\varepsilon_{it}-1}{\varepsilon_{it}}} + z_{ij}^{st\,\frac{\varepsilon_{it}-1}{\varepsilon_{it}}} \right]^{\frac{\varepsilon_{it}-1}{\varepsilon_{it}}} \qquad t = (r,s) \qquad (11)$$

The first-order condition implies that:

$$\frac{p_i^r}{p_i^s} = \left(\frac{z_{ij}^{rt}}{z_{ij}^{st}} \right)^{-\frac{1}{\varepsilon_{it}}} \qquad t = (r,s) \qquad (12)$$

Self-sufficiency rate is an important indicator in IO modeling. Using the denotation system of this paper, we could write the self-sufficiency rate of input *i* consumed by industry *j* in country t as:

$$\theta_{ij}^t = \frac{p_i^t z_{ij}^{tt}}{p_i^r z_{ij}^{rt} + p_i^s z_{ij}^{st}} \qquad t = (r,s) \qquad (13)$$

Substituting Equation (12) into Equation (13), we obtain the following:

$$\theta_{ij}^{r} = \frac{1}{1+\left(\frac{p_i^r}{p_i^s}\right)^{\varepsilon_{ir}-1}} \tag{14}$$

$$\theta_{ij}^{s} = \frac{1}{1+\left(\frac{p_i^r}{p_i^s}\right)^{1-\varepsilon_{is}}} \tag{15}$$

It is easy to find that the right hand of formula (14) has no appearance of j. So we can simplify notation by letting $\theta_i^t = \theta_{ij}^t$ for all i and j and set $p_i = \frac{p_i^r}{p_i^s}$, which yields:

$$\theta_i^r = \frac{1}{1+p_i^{\varepsilon_{ir}-1}} \tag{16}$$

$$\theta_i^s = \frac{1}{1+p_i^{1-\varepsilon_{is}}} \tag{17}$$

Based on the profit-maximizing mechanism, a product's self-sufficiency rate is no longer a fixed parameter. It is endogenous and up to three parameters: price level of product i in country r, price level of product i in country s and elasticity of substitution of products between two countries. If the price ratio is less than 1, the greater the elasticity of substitution, the higher the country's self-sufficiency rate. If the price ratio is greater than 1, the greater the elasticity of substitution, the lower the country's self-sufficiency rate. When the elasticity of substitution is greater than 1, the price ratio and a country's self-sufficiency rate show an inverse relationship. If the elasticity of substitution is smaller than or equal to 1, the country's self-sufficiency rate becomes less sensitive to changes in the price ratio.

2.2 Interregional Input-Out Model with endogenous Self-sufficiency rate

Basic equilibrium relationship of two-country interregional input-output table can be written as:

$$\sum_{j=1}^{n} x_{ij}^{rr} + \sum_{j=1}^{n} x_{ij}^{rs} + y_i^r = x_i^r \tag{18}$$

$$\sum_{j=1}^{n} x_{ij}^{sr} + \sum_{j=1}^{n} x_{ij}^{ss} + y_i^s = x_i^s \tag{19}$$

Where x_{ij} is the value of products produced by sector i used as inputs by sector j. x_i is the value of total output by sector i. y_i is the value of final demand for sector i.

Express Equation (18) and (19) as the product of physical quantity and price:

$$\sum_{j=1}^{n} p_i^r z_{ij}^{rr} + \sum_{j=1}^{n} p_i^r z_{ij}^{rs} + y_i^r = p_i^r z_i^r \tag{20}$$

$$\sum_{j=1}^{n} p_i^s z_{ij}^{sr} + \sum_{j=1}^{n} p_i^s z_{ij}^{ss} + y_i^s = p_i^s z_i^s \tag{21}$$

Here, z_{ij} should not be thought of as the actual statistical figures in interregional input-output table. From the theoretical standpoint, they could be view as optimal solutions of profit-maximizing mechanism. Therefore we can obtain the following equations from Equations (13), (16) and (17):

$$p_i^r z_{ij}^{rr} = \theta_i^r \left(p_i^r z_{ij}^{rr} + p_i^s z_{ij}^{sr}\right)$$

$$p_i^r z_{ij}^{rs} = (1-\theta_i^s)\left(p_i^r z_{ij}^{rs} + p_i^s z_{ij}^{ss}\right)$$

$$p_i^s z_{ij}^{sr} = (1-\theta_i^r)\left(p_i^r z_{ij}^{rr} + p_i^s z_{ij}^{sr}\right)$$

$$p_i^s z_{ij}^{ss} = \theta_i^s \left(p_i^r z_{ij}^{rs} + p_i^s z_{ij}^{ss}\right) \qquad (22)$$

Substituting (8) and (9) into (22), which yield:

$$p_i^r z_{ij}^{rr} = \theta_i^r p_i^{cr} z_{ij}^r = \theta_i^r a_{ij}^r p_j^r z_j^r = \theta_i^r a_{ij}^r x_j^r$$

$$p_i^r z_{ij}^{rs} = (1-\theta_i^s) p_i^{cs} z_{ij}^s = (1-\theta_i^s) a_{ij}^s p_j^s z_j^s = (1-\theta_i^s) a_{ij}^s x_j^s$$

$$p_i^s z_{ij}^{sr} = (1-\theta_i^r) p_i^{cr} z_{ij}^r = (1-\theta_i^r) a_{ij}^r p_j^r z_j^r = (1-\theta_i^r) a_{ij}^r x_j^r$$

$$p_i^s z_{ij}^{ss} = \theta_i^s p_i^{cs} z_{ij}^s = \theta_i^s a_{ij}^s p_j^s z_j^s = \theta_i^s a_{ij}^s x_j^s \qquad (23)$$

Substituting (23)，Equation (20) and (21) can be written as:

$$\sum_{j=1}^n \theta_i^r a_{ij}^r x_j^r + \sum_{j=1}^n (1-\theta_i^s) a_{ij}^s x_j^s + y_i^r = x_i^r \qquad (24)$$

$$\sum_{j=1}^n (1-\theta_i^r) a_{ij}^r x_j^r + \sum_{j=1}^n \theta_i^s a_{ij}^s x_j^s + y_i^s = x_i^s \qquad (25)$$

which can be rewritten in matrix form as:

$$A^r \hat{\theta}^r X^r + A^s\left(1-\hat{\theta}^s\right) X^s + Y^r = X^r \qquad (26)$$

$$A^r\left(1-\hat{\theta}^r\right) X^r + A^s \hat{\theta}^s X^s + Y^s = X^s \qquad (27)$$

Then in the block matrix is:

$$\begin{bmatrix} A^r \hat{\theta}^r & A^s\left(1-\hat{\theta}^s\right) \\ A^r\left(1-\hat{\theta}^r\right) & A^s \hat{\theta}^s \end{bmatrix} \begin{bmatrix} X^r \\ X^s \end{bmatrix} + \begin{bmatrix} Y^r \\ Y^s \end{bmatrix} = \begin{bmatrix} X^r \\ X^s \end{bmatrix} \qquad (28)$$

Here $\hat{\theta}^r$ and $\hat{\theta}^s$ are the diagonal matrix which typical entry denotes θ_i^r and θ_i^s respectively. A^r and A^s are the input coefficients matrix of country r and country s which typical entry denotes a_{ij}^r and a_{ij}^s defined in (9).

Doing this with matrix algebra, we can obtain the expression of the interregional input-output model with endogenous self-sufficiency rate, which could be used to estimate the impact of interregional shocks.

$$\begin{bmatrix} X^r \\ X^s \end{bmatrix} = \begin{bmatrix} I - A^r\hat{\theta}^r & -A^s(1-\hat{\theta}^s) \\ -A^r(1-\hat{\theta}^r) & I - A^s\hat{\theta}^s \end{bmatrix}^{-1} \begin{bmatrix} Y^r \\ Y^s \end{bmatrix} \qquad (29)$$

It could be expressed as in the relationship of the change in variables:

$$\begin{bmatrix} \Delta X^r \\ \Delta X^s \end{bmatrix} = \begin{bmatrix} I - A^r\hat{\theta}^r & -A^s(1-\hat{\theta}^s) \\ -A^r(1-\hat{\theta}^r) & I - A^s\hat{\theta}^s \end{bmatrix}^{-1} \begin{bmatrix} \Delta Y^r \\ \Delta Y^s \end{bmatrix} \qquad (30)$$

From equation (30), it could be noticed that the shocks from country s as the change in exports to country s from country r which reflected in ΔY^r vector may have the impact on the total output of both countries by the transmission matrix composed of self-sufficiency rates and input coefficients of both countries.

Compared to the two-country classic interregional input-output model, the transmission matrix of new model includes different parameters. By endogenizing self-sufficiency rates, we introduce the relative price level among countries and the elasticity of substitution into model. Because the two models derived from the same balance equations, they will produce identical calculation results when the elasticity of substitution and the relative price level among countries of the period of IRIO tables are exactly the same as those used in the simulation. However, compiling interregional input-output table is time-consuming so that IO tables are always historic materials, the relative price level among countries and the elasticity of substitution are not constant in fact obviously. The results calculated will differ when we use those estimated parameters whose value are actual value at the period of when the shock happened.

A main advantage of our approach is that we could introduce more accurate estimation about the relative price level among countries and the elasticity of substitution based on more timely statistical data into the model to calculate the effect of shocks to pursing the more reliable results. However, this also presents certain challenges. Although input coefficients can be obtained from the input-output table, other parameters still require external data sources, which will be discussed in detail in the next chapter.

3. Scenarios settings and parameters estimation

This section describes the setup of the shock scenarios and the parameters required for the model presented in section 2. In order to estimate the model, three types of parameters are needed to assign to values: the input coefficient matrix, the relative price level among regions and the elasticity of substitution between countries of each sector.

3.1Shocks

To check the differential implications of exogenous shock transmission between our proposed framework and the classical model, we construct a external demand shock on cross-country scenario. Our analysis focuses on the world's two largest economies, China and the United States, both of which serve as critical export markets for each other. Fluctuations in bilateral trade flows are posited to induce uncertainty-driven shocks to their respective economic systems. Accordingly, we simulate counterfactual variations in (i) the top three U.S. export sectors to China by trade volume - the manufacture of textiles, wearing apparel, and leather products; the manufacture of computer, electronic, and optical products; and the manufacture of furniture and other manufacturing products (ii) the corresponding top three Chinese export sectors to the United States- the manufacture of other transport equipment; the manufacture of motor vehicles, trailers, and semi-trailers; and the manufacture of machinery and equipment, n.e.c.

Let r represent China and s represent the United States.As described in equation (23),the impact of exports(demand-side) shocks on the value-added of China is

$$\Delta\boldsymbol{V^r} = \left[\left(I - A^r\hat{\theta}^r\right)^{-1}\Delta Y^r - \left(I - A^r\hat{\theta}^r\right)^{-\mathbf{1}}A^s\left(1-\hat{\theta}^s\right)\left(I - A^s\hat{\theta}^s\right)^{-1}\Delta Y^s\right]\ \hat{v}^r \quad (31)$$

the impact of exports(demand-side) shocks on the value-added of United States is

$$\Delta\boldsymbol{V^s} = \left[\left(I - A^s\hat{\theta}^s\right)^{-1}\Delta Y^s - \left(I - A^r\hat{\theta}^r\right)^{-\mathbf{1}}A^r\left(1-\hat{\theta}^r\right)\left(I - A^s\hat{\theta}^s\right)^{-1}\Delta Y^r\right]\hat{v}^s \quad (32)$$

Then, we obtain the change in value added, which is equal to the change in total output multiplied by the value-added rate$\hat{v}$.

According equation(24), We calculated the estimated results of the impact of demand-side shocks in the classic interregional input-output model. That is

$$\Delta\boldsymbol{V^r} = [(I - A^{rr})^{-\mathbf{1}}\Delta\boldsymbol{Y^r} - (\boldsymbol{I} - \boldsymbol{A^{rr}})^{-\mathbf{1}}\boldsymbol{A^{rs}}(\boldsymbol{I} - \boldsymbol{A^{ss}})^{-\mathbf{1}}\Delta\boldsymbol{Y^s}]\ \hat{v}^r$$
$$\Delta\boldsymbol{V^s} = [(I - A^{ss})^{-\mathbf{1}}\Delta\boldsymbol{Y^s} - (I - A^{ss})^{-\mathbf{1}}A^{rs}(I - A^{rr})^{-\mathbf{1}}\Delta\boldsymbol{Y^r}]\,\hat{v}^s \quad (33)$$

3.2 The Estimation of Parameters

Taking data from the World Input-Output Database 2014, we have compiled a three-country input-output table that includes China, the United States, and the rest of the world(ROW). The sector classification is unchanged. We can derive the input matrix from the three-country input-output table. In estimating the two-country model, we exclude data for the 'rest of the world' (ROW). While this may introduce some bias, the comparative nature of the study suggests that its impact on the results is unlikely to be substantial. The two additional categories of parameters required by the model are not directly obtained in WIOD table, necessitating a detailed exposition here.

(1)The Estimation of Relative Price Level among Countries.

The relative price levels between sectors across countries are not easily estimated with precision. Existing statistical data and research have identified two main indicators in this context. One is the Terms of Trade, commonly used in trade studies, and the other is the purchasing power parity (PPP) index, constructed by the United Nations ICP (International Comparison Program) for international comparisons. Terms of trade are typically measured as the ratio of the import price index to the export price index. While this indicator allows for sectoral results, it is based on price indices and thus only reflects the relative changes in import and export prices, rather than the relative price levels that we require. Similar issues arise with measures of relative price levels constructed using the ratio of the import price index to the CPI or PPI (Tian et al., 2021). The purchasing power parity index, on the other hand, is a country-specific price index built on the expenditure approach to GDP accounting. Economically, this indicator aligns with the theoretical concept of relative price levels. Although the data published by the ICP project does not provide sector-level relative price measures, it is possible to estimate sectoral relative price levels by matching the PPP indices of various goods to corresponding industries. The classification of products in PPP and their corresponding industry sectors are matched with the intermediate input sectors in the WIOD table, as shown in the Table 1.

Table 1 Identification of Relative Price Level by industry

PPPs by Product (ICP)	Industry (WIOD)	Relative Price Level
MEAT	1,2,7,8	5,30
FISH AND SEAFOOD	3	5,32
GROSS DOMESTIC PROUCT	4	4,18
FOOD AND NON-ALCOHOLIC BEVERAGES	5	6,33
CLOTHING AND FOOTWEAR	6	7,40
COMMUNICATION	9	2,98
MACHINERY AND EQUIPMENT	10,11,12,13,14,15,16,17, 18,19	9,33
PURCHASE OF VEHICLES	20,21	5,53
FURNISHINGS, HOUSEHOLD EQUIPMENT AND ROUTINE HOUSEHOLD MAINTENANCE	22	6,43
ACTUAL HOUSING, WATER, ELECTRICITY, GAS AND OTHER FUELS	23,24,25,26	2,87
CONSTRUCTION	27	2,31
PURCHASE OF VEHICLES	28,29,30	5,53
TRANSPORT	31,32,33,34,35	4,77
RESTAURANTS AND HOTELS	36	3,98
ACTUAL RECREATION AND CULTURE	37	3,97
COMMUNICATION	38,39,40	2,98
ACTUAL MISCELLANEOUS GOODS AND SERVICES	41,42,43,44,45,46,47,48	4,78
ACTUAL EDUCATION	49,50,51,52	2,91
ACTUAL HEALTH	53	2,32

ACTUAL MISCELLANEOUS GOODS AND SERVICES	54,55	4,78
1113000:NET PURCHASES ABROAD	56	6,77

(2)**The estimation of Substitution Elasticity**.

In this paper, the substitution elasticity between domestic and foreign goods is defined as the Armington elasticity (Armington ,1969).The literature on substitution elasticity estimation has accumulated a large body of work(Soderbery, ,2015; Feenstra. et al,2018; Anderson and Yotov, 2021; Caliendo and Parro, 2022).However, the estimated values of substitution elasticity are highly sensitive to the chosen technological specification, with different production functions potentially leading to significant differences in empirical results.

This study focuses on the bilateral relationship between China and the United States, requiring industry-level Armington elasticities for China's exports to the U.S. and the U.S. exports to China. Therefore, we collected the range of Armington elasticities for China's exports to the U.S. and for the U.S.'s exports to China. Hertel, T. et al. (2007) use the GTAP database to provide elasticities for China's manufacturing sectors in relation to the U.S. Additionally, I supplement this with the substitution elasticities for other sectors estimated by Shi, W. (2010). Feenstra, R. C., & Romalis, J. (2014) calculate the substitution elasticities for China's manufacturing sectors based on U.S. import data, and I complement this with the estimates for other sectors from Caliendo, L., & Parro, F. (2015).

4.Simulation Results

Based on the scenarios settings and parameters estimation provided in Section 3, this section describes the simulation results of export shock and compare the differences between the results of the two models. While this may introduce some bias, the comparative nature of the study suggests that its impact on the results is unlikely to be substantial.

4.1 Results

The result of the exports shock from China to United State, transmitted to the value added of both China and the United States, is shown in Figure 1. The figure illustrates that changes of exports for China's manufacture of textiles, wearing apparel, and leather products (sector 6); the manufacture of computer, electronic, and optical products (sector 17); and the manufacture of furniture and other manufacturing products (sector 22) will drive domestic overall value added and indirectly promote the growth of value added in the United States. We found that for every one million dollars change in

China's exports to United States, both models estimate a deviation of over 0.1 million dollars in the change in value-added.

Fig 1The Impact of China Export Shock

The result of the export shock from United States to China is shown in Figure 2. The simulation results of the two models presented in the figure are nearly identical. An increase in the exports of the manufacture of other transport equipment (sector 21), the manufacture of motor vehicles, trailers, and semi-trailers (sector 20), and the manufacture of machinery and equipment, n.e.c. (sector 19) will drive the growth of value added in the United States, while the impact on China's value added is less than 0.1.

Fig 2 The impact of United States Export Demand Shock

According to the model setup, if the implied self-sufficiency rate in the data from the WIOD is the same as the model parameters, the results will be the same. The current results indicate that the data from the WIOD differ from the parameters set in the simulation scenario. This is because input-output tables are often compiled using historical data, which introduces a time lag compared to the present.

Observing the U.S. export shock, we find that although the IRIO-SSR still estimates lower value added for China and higher value added for the U.S., the simulation results are nearly identical. This suggests that the current settings for elasticity of substitution and price indices are very close to those in the input-output table.

That means that in previous estimations, we may have overestimated the impact of the increase in China's exports on China's value added, while underestimating its effect on the United States.

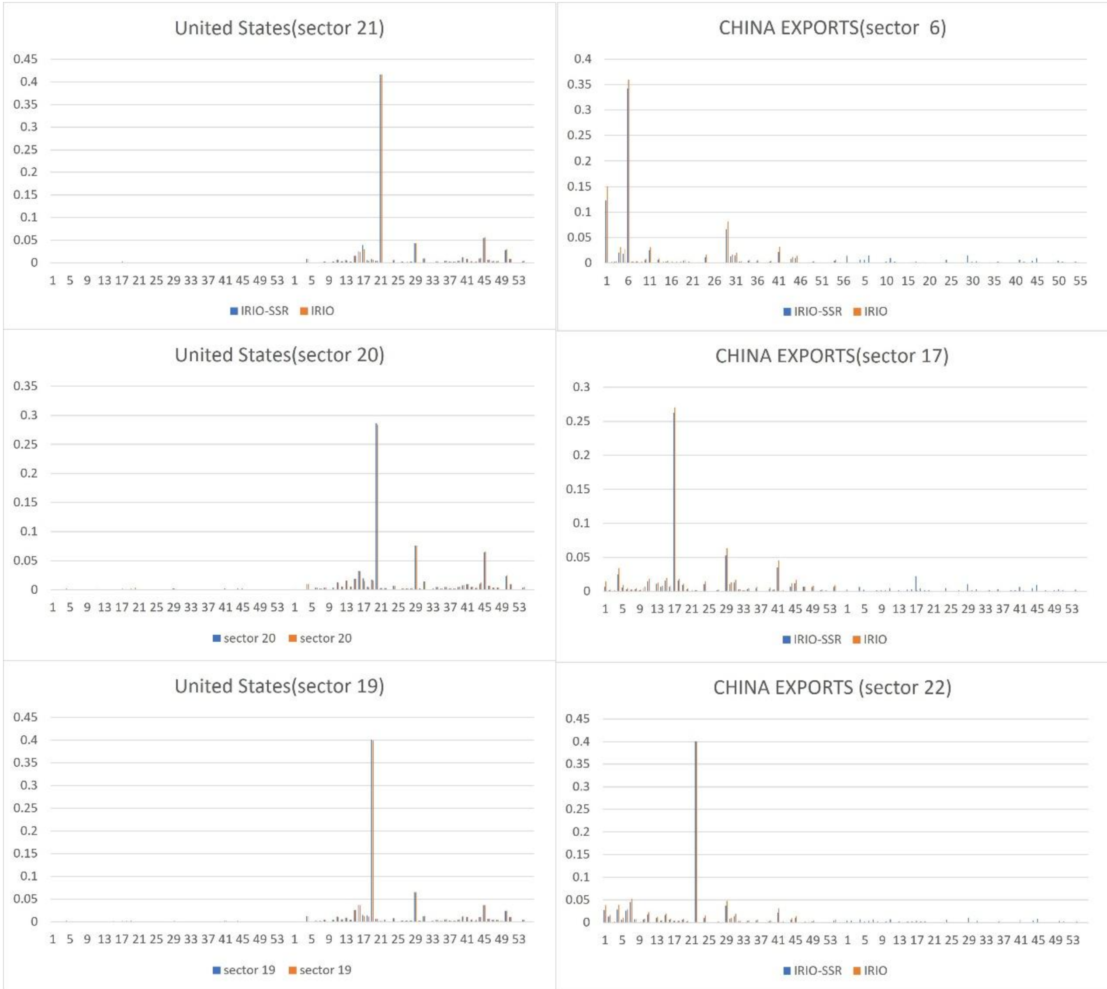

Fig 3 industry level export demand shock

Furthermore, Figure 3 shows the results of the impact of export shock at the industry level. In China, with the increase in export demand, production in the three major industries will be significantly stimulated. This stimulus effect is not limited to the industries that directly benefit but extends to upstream and downstream industries closely related to these sectors. For example, China's export effects will notably boost production in domestic sectors 29, 30, 31, and 41. That implies that not only the manufacturing industry's own production activities are enhanced, but also that the related supply chain and industrial chain, both upstream and downstream, will grow, further driving overall economic growth.

This transmission mechanism of export effects indicates that the increase in export demand not only directly raises the output of the relevant industries but also stimulates the overall growth of upstream and downstream sectors through inter-industry linkages. This multi-dimensional impact is crucial for understanding the long-term driving effects of export growth on the economy and industrial structure.

4.2 Other scenario

Although the elasticity of substitution is often treated as a parameter in short-term analyses, in reality, it changes over time due to factors such as technological progress. The value of the elasticity of substitution determines the relationship between import and domestic products, thereby affecting the self-sufficiency rate.

The question arises whether the elasticity of substitution will change the nature of the model and, in turn, influence the results of shock transmission. Therefore, we consider two extra scenarios for the elasticity of substitution and assess its impact on the model. First, assuming a substitution elasticity of 1, the self-sufficiency rate of all industries will be fixed at 50%, with the ratio of domestic to imported goods unaffected by price changes. Second, we assume that the substitution elasticity is the same for both countries, and that the elasticity is uniform across all products. According to the formula, it can be inferred that a higher substitution elasticity leads to a lower self-sufficiency rate. We set the substitution elasticity between Chinese domestic and imported goods, as well as between U.S. domestic and imported product at 5, and then adjust it to 2. The results are presented in Table 2.

Table 2 the results of impact of external demand shock

value-added	type	Substitution elasticity	6	17	22	21	20	19
			CHN			U.S.		
CHN	IRIO		0.89	0.70	0.88	0.02	0.03	0.02
U.S.			0.01	0.01	0.00	0.78	0.73	0.80
CHN		1	1.15	1.03	1.00	0.33	0.42	0.34
U.S.		1	0.54	0.42	0.40	0.93	1.03	0.95
CHN	IRIO-SSR	(Section3.2)	0.74	0.58	0.76	0.02	0.02	0.01
U.S.		(Section3.2)	0.15	0.12	0.12	0.78	0.73	0.81
CHN		5	0.86	0.66	0.85	0.36	0.46	0.39
U.S.		5	0.03	0.05	0.03	0.42	0.29	0.42

CHN	2	0.74	0.58	0.76	0.02	0.02	0.01
U.S.	2	0.15	0.12	0.12	0.78	0.73	0.81

5.Conlusions

Classic input-output models often assume that domestic and imported products are either perfectly substitutable or non-substitutable in measuring the effects of external demand shocks. However, these assumptions do not fully reflect real-world trade patterns. My research develops an interregional input-output model with an endogenous self-sufficiency rate, offering a more nuanced approach to measuring the effects of external demand shocks. After two stages of optimization decisions, the economic agents incorporate key parameters, such as relative price level among the countries and substitution elasticities, into the transmission matrix. Endogenizing the self-sufficiency rate allows a country's self-sufficiency to vary with changes in relative price level among the countries and substitution elasticities. And introducing the Armington specification that permits a non-zero substitution elasticity between imported and domestic products in Leontief inverse, addressing the limitations of classic methodologies. To compare the differences between the classical model and our proposed model in transmitting exogenous shocks, we have set up scenarios involving changes in representative sectors in China's exports to the U.S. and the U.S.'s exports to China. The information of the relative price level among countries comes from the cross-country price indices of the United Nations International Comparison Program((ICP) and the distribution range of the elasticity of substitution between domestic and imported products from the existing empirical results of the Armington elasticity.

However, the current model remains a highly simplified theoretical framework. s and prices. We plan to undertake this and other extensions in our future. First, the existing framework assumes that the same product has identical prices in both countries. In reality, however, due to factors such as tariffs, the price system is more complex. This model, when extended to incorporate a more nuanced price system, can also provide a theoretical framework for studying the impact of tariff policies. Second, the current accuracy of the relative price level among the countries matching is limited. We plan to incorporate customs data to further optimize the industry-level relative price level among the countries.Third, there is significant variation in the estimation of Armington elasticities. Once the model is expanded to a multi-country framework, it will become more complex, and the parameter setting will include certain noise and biases that need to be addressed.

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