On the Role of Profits-Wages Ratios in the Determination of the Long-Run Behavior of International Relative Prices

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Abstract

This paper reconstructs and evaluates the theory of international relative prices (IRP) based on the theory of 'real competition'. The main thesis of the theory is that the long-run of the IRP of tradable commodity bundles is determined exclusively by their relative total unit labor costs (RTULC). This is equivalent to the proposition that the total profits-wages ratios (TPWR) of these two bundles are sufficiently similar across time and, therefore, neutral in the long run. We identified a set of problems that questions the strength of the theory. Firstly, due to accounting reasons, the proposed hypotheses cannot constrain IRP to depend solely on the RTULC. Secondly, the theoretical and empirical arguments advanced to constrain the TPWR are weak. The paper conducts a large-scale study of industries' TPWR and finds that their statistical regularities do not support the constraints necessary for the validity of the theory's main thesis.

Keywords: real exchange rates; terms of trade; absolute advantage; unit labor costs; profit-wages ratios; capital intensities.
JEL Class: B51; C67; D57; D33; E25.

1 Introduction

The link between the international relative prices (IRP) of commodities and their relative unit labor costs is of paramount importance in international economics, particularly for the study of the competitiveness and trade of nations. The IRP can be the terms of trade (ToT) between two commodity bundles or the bilateral or effective real exchange rates (RER) of a nation. Some authors sustain that the long-run behavior of IRP *depends* on its associated relative unit labor costs whereas others *define* the equilibrium IRP as these relative costs. These two views have originated from both neoclassical and critical traditions in economic theory. Whereas the former is based on the purchasing power parity (PPP) hypothesis and the principle of comparative advantage,[1] the latter originate from the principle of absolute advantages (PAA).[2]

One of the most applied theories of IRP based on the PAA is the one developed by Shaikh (1980; 1991; 1995; 1999; 2016, ch. 11), which combines his work on 'real competition' (Shaikh, 2016, ch. 7) and on the labor theory of value and industrial prices (Shaikh, 1984; 1998; 2012; 2016, ch. 9).[3] The main thesis of Shaikh's theory is that the long-run behavior, or center of gravity, of the IRP of any pair of tradable commodity bundles, is determined by their relative *total* unit labor costs (RTULC).[4] It is argued that while forces of competition are sufficiently strong to generate relative market prices to gravitate around relative production prices, i.e., prices that generate uniform profit rates, relative production prices have as backbone the RTULC. As we shall see in Section 2, for any pair of tradable commodity bundles, produced domestically or internationally, their relative prices equal their RTULC if and only if their ratios of total profits to total wages are equal. Hence, the thesis on the long-run de-

[1] Krueger (1983, ch. 2) and Isard (1995, ch. 4 & 6) provides a thorough exposition of IRP based on the neoclassical tradition. Officer (1976, p. 10-3), Marsh and Tokarick (1994), and Hinkle and Nsengiyumva (1999) provide a survey where, based on cost parity approach to PPP, unit labor costs are used as a measure of competitiveness and of the degree of misalignment of the RER.

[2] See, inter alia, Shaikh (1979, 1980), Dosi et al. (1990), and Bellino and Fratini (2022).

[3] Other approaches within the PAA use linear production models to extend the Sraffian model of production prices to international trade –see, e.g., Parrinello (2009), Vasudevan (2012), and Bellino and Fratini (2022) and the literature cited there. Other studies base their approach on Pasinetti's (1981) structural dynamics approach, like in Garbellini (2021) and Machado and Trigg (2021). All these approaches consider prices to be the most important element in firms' or industries' competitiveness. However, there are other approaches within the PAA, like the literature on technology-gap (see, for instance, Milberg 1994), which consider that firms' competitive position depends on other variables in addition to prices. Aglietta and Oudiz (1984) define the RER with the relative unit labor costs and build a model to explain their long-run determinants.

[4] 'Total' refers to the direct and indirect labor costs, that is, the vertically integrated labor cost.

termination of IRP by its RTULC is equivalent to the thesis of the long-run neutrality of the relative total profits-wages ratios (RTPWR). To obtain this result, the literature proposes theoretical and empirical arguments making the RTPWR of industries *within countries* a disturbance term centered around one, expecting with this to constrain the RTPWR of the *internationally* produced commodities.

The literature based on Shaikh's theory reports empirical evidence in favor of the hypothesis of a *long-run association between IRP and RTULC* for different economies —developed and underdeveloped economies and before and after the Bretton Woods system collapse— and for different indicators of IRP, such as bilateral real exchange rates,[5] the real effective exchange rates –i.e., the RER of one country with its multiple trading partners–,[6] and the terms of trade between pairs of commodities.[7] The regression exercises typically include control variables (such as government expenditure and GDP), other variables capturing short-term determinants of IRP (e.g., capital flows and interest and profit rates differentials), and a factor that controls the non-tradable commodities included in the data. Due to data availability, sometimes the evidence involves the direct, instead of the total unit labor costs. In this way, the reported evidence suggests that this long-run association between IRP and their relative unit labor costs can be considered a stylized fact of modern economies.

In our reconstruction and evaluation of Shaikh's theory, we have identified many problems with its foundations. The main one is that its first hypothesis *is not efficient* producing the proposed thesis. The hypothesis states that in the long run and *within a country*, relative prices between any two commodities equal their RTULC. This is equivalent to the constraint of homogeneity in total profits-wages ratios *within a country*. Hence, one first problem of the thesis that the long-run behavior of IRP is determined exclusively by the RTULC is that requires *international* homogeneity of total profits-wages ratios. This needed (as opposed to proposed) hypothesis contradicts the criticisms of this literature of the neoclassical assumptions of homogeneity in the techno-distributive conditions of nations, describing them as unrealistic. We shall show that even if we assume within-country uniformity in total profits wage ratios, still the relative wage shares of the two countries affect the long-run IRP.

[5] See Boundi-Chraki (2017), Martínez-Hernández (2010), Moreno-Rivas (2018), Ruiz-Nápoles (2001, 2010, 2023), Shaikh (2002), and Shaikh and Antonopoulos (2012).

[6] See Antonopoulos (1999), Ersoy (2010), Góchez-Sevilla and Tablas (2013), Kvangraven (2018, ch. 2), Martínez-Hernández (2017), Papafragkos (2023), Poulakis and Tsaliki (2023a,b), Ruiz-Nápoles (2010), and Stravelakis (2022).

[7] See Boundi-Chraki (2019, 2020) and Boundi-Chraki and Perrotini-Hernández (2021).

Secondly, the arguments advanced by the literature to sustain their proposed (but also needed) hypotheses are *weak*. The *theoretical* arguments, such as the existence of strong inter-industry connections, constitute speculations because they do not prove they can constrain the national nor international RTPWR to be one. As for the *empirical* arguments, the weakness is more serious: there is a vacuum of knowledge on the empirical properties of national and international profit-wages ratios in the literature. Hence, not only there is an absence of empirical evidence to sustain the hypotheses, but also the empirical arguments used in the literature are unwarranted.

The paper contributes to addressing this vacuum by conducting the first large-scale study of industries' profits-wages ratios by studying the WIOD database (42 countries during 2000-2014 for up to 56 industries). It is shown that the statistical patterns of the total profits-wages ratios question the required constraints necessary for Shaikh's thesis. The ratios display a remarkable persistence in their empirical distribution, showing a statistical tendency to cluster around central values, with limited variability and asymmetry. However, the remaining variability of the ratios *within countries* and the variability of their central tendencies *across countries* makes difficult to sustain that the total profits-wages ratios of any two industries will be sufficiently close for IRP to be determined in the long-run *exclusively* by its RTULC. Hence, *there is evidence that the total profit-wages ratios are not neutral in the long-run determination of IRP*.

Shaikh and his followers sustain that their theory is based either on the classical or 'the real' theory of competition. However, none of the features from these theories can constrain the RTPWR of any two pairs of domestic or international commodity baskets in such a way to make them neutral in the long-run determination of relative prices. In the best case, they can generate world uniformity in the wage rate and the profit rate, but capital intensities can still be heterogeneous.

The rest of the paper is composed of the following sections. Section 2 constructs an economy where we can (i) study the *accounting structure* of national and international relative prices at market and production prices and (ii) represent Shaikh's theory of IRP. Section 3 introduces the theory of 'real competition' and the hypotheses and arguments proposed to derive the theory's main thesis. After that, it evaluates the effectiveness of the hypotheses to produce the results of the theory as well as the theoretical and empirical foundations of these hypotheses. Section 4 studies the statistical properties of industries' profits-wages ratios and reports the lack of evidence for the needed hypotheses. Section 5 concludes developing some implications for related literature.

2 National and International Relative Prices: Accounting Structure and Production Prices

2.1 A general model of capitalistic market economy

Suppose that the world economy is composed of two countries, A and B,[8] each one with its own currency, fulfilling the following assumptions. Countries have capitalist economies with no government and produce n divisible commodities by single-product industries. Land is abundant and labor is indispensable, so value-added is positive in each industry. Let N be the set of all labels of commodities produced $N = \{1, 2, \ldots, n\}$. The price of commodity $j \in N$ is uniform across buyers and sellers —the law of one price holds. *All commodities are tradable but there is no trade in commodities used as means of production.*[9] There are no assumptions on the input-output relations, the nature of the labor inputs, the composition of the capital advanced, and the rates of return to workers and capital, except that capitals operating within each industry use the same technology. All variables are functions of time.

2.2 Relative market prices within countries

The cost-of-production decomposition of the value of the output of industry j is

$$x_j = \text{wages}_j + \text{profits}_j + \text{value of means of production}_j. \tag{1}$$

Given that $x_j = p_j q_j$, where p_j is the market price of the j-th commodity and q_j its quantity produced, p_j can be decomposed as

$$p_j = \omega_j + \pi_j + \lambda_j \quad \text{for} \quad j \in N, \tag{2}$$

where $\omega_j \equiv \frac{\text{wages}_j}{q_j} > 0$, $\pi_j \equiv \frac{\text{profits}_j}{q_j} \geqslant 0$, and $\lambda_j \equiv \frac{\text{means of production}_j}{q_j} \geqslant 0$ represent the labor costs, profits, and the means of production costs in industry j, all of them per unit of output j. Because ω_j and π_j represent the unitary wages and profits for industry

[8] When studying one country in isolation, we drop off the superscripts A and B and only include them when we need to differentiate information from both economies.

[9] We acknowledge that the study of IRP without imported means of production is too restrictive but this assumption is needed to represent Shaikh's theory in its purest form, i.e., without corrections for non-tradable commodities and with no imported means of production. One way to accommodate this assumption is that countries produce in autarchy and are suddenly open to free trade. On this see also (Martínez-Hernández, 2017, fn 3, p. 7)

j, we will refer to $\psi_j \equiv \frac{\pi_j}{\omega_j} \geqslant 0$ for $j \in N$ as the *direct* profits-wages ratios.

From the Smithian value decomposition[10] we can conduct the vertical integration of the value-added in λ_j, i.e., aggregate the wages and profits in the value chain of one unit of commodity j, and decompose $\lambda_j = \omega_j^I + \pi_j^I$, where ω_j^I and π_j^I are the *indirect* labor costs and profits per unit of commodity j. Appendix A.1 provides a formal presentation. Hence, (2) can be expressed *equivalently* as:

$$p_j = \Omega_j + \Pi_j = \Omega_j(1 + \Psi_j) \quad \text{for} \quad j \in N, \tag{3}$$

where $\Omega_j \equiv \omega_j + \omega_j^I$ is the *total* or vertically integrated unit labor costs, $\Pi_j \equiv \pi_j + \pi_j^I$ are the total unit profits, and $\Psi_j \equiv \frac{\Pi_j}{\Omega_j}$ are the total profits-wages ratios, the most important variable in this paper.

We can express every Ψ_j and $(1 + \Psi_j)$ as the weighted average of the direct profits-wages ratios of all industries, ψ_j:

$$\Psi_j = \sum_{i=1}^{n} \psi_i m_{ij} \quad \text{for} \quad j \in N, \tag{4}$$

$$(1 + \Psi_j) = (1 + \sum_{i=1}^{n} \psi_i m_{ij}) = \sum_{i=1}^{n}(m_{ij} + \psi_i m_{ij}) = \sum_{i=1}^{n}(1 + \psi_i)m_{ij} \quad \text{for} \quad j \in N, \tag{5}$$

where $m_{ij} \geqslant 0$ for $i, j \in N$ are the weights and $\sum_{i=1}^{n} m_{ij} = 1$. This weighting system condenses the process of vertical integration mentioned above. Equations (4)-(5) show the relevance for each total profits-wages ratio Ψ_j of the properties of all the direct profits-wages ratios ψ_j and weights m_{ij} as well as their interaction. Equation (4) is an alternative decomposition to that of Shaikh (1984, p. 65-9). Appendices A.2 and A.3 provide a detail presentation and comparison of both decompositions, whereas A.4 provide some statistical properties.

From equation (3) we can express relative prices of commodities j and k as

$$\frac{p_j}{p_k} = \frac{\Omega_j}{\Omega_k} \cdot \frac{(1 + \Psi_j)}{(1 + \Psi_k)} \quad \text{for} \quad j, k \in N, \tag{6}$$

where $\frac{p_j}{p_k}$ are the national relative prices (NRP), $\frac{\Omega_j}{\Omega_k}$ are the relative total unit labor costs (RTULC) and $\frac{(1+\Psi_j)}{(1+\Psi_k)}$ are the relative total profits-wages ratios (RTPWR). Shaikh (2016, p. 386) calls (6) the fundamental equation of relative prices. However, notice that price expressions (2) and (3) are *equivalent*, i.e„ they contain the same information because no additional constraints have been imposed.

[10] See Smith (1994, ch. VI), Pasinetti (1973, §4), and Shaikh (1984, §IV).

Let us now decompose Ψ_j in terms of an economy-wide profit-wage relation and the industry-level deviations around it. The economy-wide profits-wages ratio is[11]

$$\psi \equiv \frac{\text{Total profits}}{\text{Total wages}} = \frac{\sum_{j=1}^{n} \text{Profits}_j}{\sum_{j=1}^{n} \text{Wages}_j}. \tag{7}$$

Expression $(1+\psi_j) = \frac{\text{Wages}_j + \text{Profits}_j}{\text{Wages}_j} = \frac{\text{Value Added}_j}{\text{Wages}_j}$ is the j-th wage-share, so $(1+\psi)^{-1}$ is the economy-wide wage-share. Define the normalized $(1+\psi_j)$ as:

$$\sigma_j \equiv \frac{1+\psi_j}{1+\psi} \quad \text{for} \quad j \in N. \tag{8}$$

If every industry has the same ψ_j, then $\psi_j = \psi$ and $\sigma_j = 1$ for $j \in N$. Based on definition (8), we can decompose (5) as:

$$(1+\Psi_j) = \sum_{i=1}^{n} (1+\psi_i) m_{ij} = (1+\psi) \sum_{i=1}^{n} \sigma_i m_{ij} \quad \text{for} \quad j \in N. \tag{9}$$

Uniformity in total profits-wages ratios, i.e., $\Psi_j = \Psi_k$ for $j,k \in N$, implies that $\sum_{i=1}^{n} \sigma_i m_{ij} = \sum_{i=1}^{n} \sigma_i m_{ik} = \sigma$.

With this decomposition, we can express relative prices (6) as

$$\frac{p_j}{p_k} = \frac{\Omega_j}{\Omega_k} \cdot \frac{(1+\psi)}{(1+\psi)} \cdot \frac{\sum_{i=1}^{n} \sigma_i m_{ij}}{\sum_{i=1}^{n} \sigma_i m_{ik}} \quad \text{for} \quad j,k \in N. \tag{10}$$

Of course, within one particular economy each price has the same $(1+\psi)$, so $\frac{p_j}{p_k} = \frac{\Omega_j}{\Omega_k} \frac{\sum_{i=1}^{n} \sigma_i m_{ij}}{\sum_{i=1}^{n} \sigma_i m_{ik}}$. However, the importance of the explicit identification of the nation wide $(1+\psi)$ in (10) will payoff for the study of IRP.

The following proposition states the necessary and sufficient conditions for $\frac{p_j}{p_k} = \frac{\Omega_j}{\Omega_k}$:

Proposition 1 *Relative prices between two commodities within an economy equal their relative total unit labor costs,*

$$\frac{p_j}{p_k} = \frac{\Omega_j}{\Omega_k} \quad \textit{for} \quad k,j \in N, \tag{11}$$

if and only if industries' total profits-wages ratios are equal, $\Psi_j = \Psi_k = \Psi$ for $j,k \in N$.

The proof is evident, but its meaning carries important implications for the explanation of IRP. First and most importantly, we need to stress that a *general* theory of relative prices equal to their RTULC results from a constraint in the relative total profits-wages ratios, not from the *special* case of zero profits in the economy, $\Psi_j = 0$ for $j \in N$, which produces $p_j = \Omega_j$, and, therefore, (11). Secondly, the loose constraints on the m_{ij}'s

[11] Let $\bar{\psi} \equiv \frac{1}{n} \sum_{i=1}^{n} \psi_i$ be the simple average of the ψ_j of all industries. In general $\psi \neq \bar{\psi}$.

make possible to find a weighting system for Proposition 1 to hold (for $\Psi_j = \Psi_k = \Psi$) even under heterogeneous *direct* profits-wages ratios. Therefore, uniformity in *direct* profits-wages ratios of all industries $\psi_j = \psi$ are *sufficient* conditions for (11).[12] Finally, notice that if profits in each industry are also positive, then $p_j = \Pi_j(1 + \Psi_j^{-1})$ and

$$\frac{p_j}{p_k} = \frac{\Pi_j}{\Pi_k} \cdot \frac{(1 + \Psi_j^{-1})}{(1 + \Psi_j^{-1})} \quad \text{for} \quad j, k \in N. \tag{12}$$

Hence, the same constraint characterizing Proposition 1, i.e., $\Psi_j = \Psi_k = \Psi$, also yields a theory of NRP governed by relative total unit profits, $\frac{p_j}{p_k} = \frac{\Pi_j}{\Pi_k}$.

2.3 Market terms of trade between two commodities

Let us distinguish the variables from the price system introduced in Section 2.2 for country $\alpha = A, B$. Consider the (net-barter) terms of trade (ToT) between commodities j and k, for $j, k \in N$, exported by A and B:

$$\frac{p_j^A}{p_k^B} = \frac{\Omega_j^A + \Pi_j^A}{\Omega_k^B + \Pi_k^B} = \frac{\Omega_j^A}{\Omega_k^B} \cdot \frac{(1 + \Psi_j^A)}{(1 + \Psi_k^B)} = \frac{\Omega_j^A}{\Omega_k^B} \cdot \frac{1 + \psi^A}{1 + \psi^B} \cdot \frac{\sum_{i=1}^n \sigma_i^A m_{ij}^A}{\sum_{i=1}^n \sigma_i^B m_{ik}^B}, \tag{13}$$

where $\frac{\Omega_j^A}{\Omega_j^B}$ are the RTULC and $\frac{(1+\Psi_j^A)}{(1+\Psi_k^B)}$ are the RTPWR between commodities j and k produced by country A and B. In contrast with NRP (10), the ToT (13) involve information of all industries between *two different countries*. That is, $\frac{(1+\Psi_j^A)}{(1+\Psi_k^B)}$ in (13), in contrast with $\frac{(1+\Psi_j)}{(1+\Psi_k)}$ in (6), refers to technical-distributive features between country A and B: (1) their economy-wide profit-wages ratios $\frac{1+\psi^A}{1+\psi^B}$ and (2) the variability of the industry-level direct profits-wages ratios σ_i^α and weights m_{ij}^α.

The following proposition states the necessary and sufficient conditions for $\frac{p_j^A}{p_k^B} = \frac{\Omega_j^A}{\Omega_j^B}$:

Proposition 2 *The terms of trade between commodity j and k exported by countries A and B equal the relative total unit labor costs,*

$$\frac{p_j^A}{p_k^B} = \frac{\Omega_j^A}{\Omega_j^B} \quad \textit{for} \quad j, k \in N, \tag{14}$$

if and only if the relative total profits-wages ratios of commodity j in country A and commodity k in country B are the same, $\Psi_j^A = \Psi_k^B$ for $j, k \in N$.

[12] This result contrasts with the case of linear production models, where $\frac{p_j}{p_k} = \frac{\Omega_j}{\Omega_k}$ if and only if there is uniform *direct* profits-wages ratios in every industry (e.g., Kurz and Salvadori, 1995, p. 112-113). Notice also that because (11) does not require $\psi_j = \psi$, then it follows that $\Psi_j = \Psi$ in general does not have to equal the economy-wide profits-wages ratio ψ in (7). For $\Psi = \psi$ we need $\sum_{i=1}^n \sigma_i m_{ij} = 1$ for $j \in N$ in (5). Sufficient conditions are $\psi_j = \psi$.

Although Proposition 2 is evident, its implications are important because, as we shall see in Section 3.2, the literature on Shaikh's theory of IRP has overlook the fact that (14) requires the equality of two magnitudes which involve technical-distributional aspects *from two different economies*. Even if we assume within-country uniformity in the total profits-wages ratios (so $\sum_{i=1}^{n} \sigma_i^\alpha m_{ij}^\alpha = \sigma^\alpha$ for $j \in N$ and $\alpha = A, B$), in general

$$\left.\frac{p_j^A}{p_k^B}\right|_{\Psi_j^B=\Psi^B}^{\Psi_j^A=\Psi^A} = \frac{\Omega_j^A}{\Omega_j^B} \cdot \frac{(1+\Psi^A)}{(1+\Psi^B)} = \frac{\Omega_j^A}{\Omega_j^B} \cdot \frac{(1+\psi^A)}{(1+\psi^B)} \cdot \frac{\sigma^A}{\sigma^B}, \tag{15}$$

where $\sum_{i=1}^{n} \sigma_i^\alpha m_{ik}^\alpha = \sigma^\alpha$. This leads to the following particular case:[13]

Proposition 3 *Suppose uniform direct profits-wages ratios within countries,* $\psi_j^A = \psi^A$ *and* $\psi_j^B = \psi^B$*. Then,*

$$\left.\frac{p_j^A}{p_k^B}\right|_{\psi_j^B=\psi^B}^{\psi_j^A=\psi^A} = \frac{\Omega_j^A}{\Omega_j^B} \tag{16}$$

if and only if the wage-shares in country A *and* B *are equal,* $1+\psi^A = 1+\psi^B$.

The most important implication of this section is that if Proposition 2 holds for any pair of (tradable) commodities produced in A and B, then it is implied that $\Psi_j^A = \Psi_k^B = \Psi$ for $j, k \in N$, that is, it implies *international uniformity in the total profits-wages ratios.*

2.4 The market (bilateral) real exchange rate

To study the TOT between nations we need to consider the bundles of commodities involved in their trade. Let $d^A = \{d_1^A, \ldots, d_n^A\}$ the export basket of country A to country B and $d^B = \{d_1^B, \ldots, d_n^B\}$ the opposite, where $d_j^\alpha \geqslant 0$ but $d_j^\alpha > 0$ at least for some $j \in N$ for $\alpha = A, B$. With these bundles let us construct an *additive index of relative prices*, which is a generalization of the ToT discussed in Section 2.3.[14] The production of these baskets generates a stream of total unit labor costs $\Omega_d^\alpha \equiv \sum_{j=1}^{n} \Omega_j^\alpha d_j^\alpha$, of total unit profits $\Pi_d^\alpha \equiv \sum_{j=1}^{n} \Pi_j^\alpha d_j^\alpha$, and has the associated total profits-wages ratios $\Psi_d^\alpha \equiv \frac{\Pi_d^\alpha}{\Omega_d^\alpha}$. With these, export price indices in country $\alpha = A, B$ are then:

$$P_d^\alpha \equiv \sum_{j=1}^{n} p_j^\alpha d_j^A = \Omega_d^A(1+\Psi_d^\alpha) = (1+\psi^\alpha) \cdot \sum_{j=1}^{n} \left(\Omega_j^\alpha \cdot d_j^\alpha \cdot \sum_{i=1}^{n} \sigma_i^\alpha m_{ij}^\alpha \right) \tag{17}$$

[13] Remember that $\psi_i = \psi$ for $i \in N$ implies that $\sigma_i = 1$ and $\sum_{i=1}^{n} \sigma_i m_{ij} = \sigma = 1$ for $j, k \in N$.
[14] When $d^A = \{0, \ldots, d_j^A, \ldots, 0\}$, $d^B = \{0, \ldots, d_k^B, \ldots, 0\}$, and $d_j^A = d_k^B = 1$.

Let e be the exchange rate between country A and B, the units of which are $\frac{€}{\$}$. If we want to express all values in terms of the currency of country B (in €), then we have to multiply all values in country A by e.[15] Therefore, the RER is then

$$\frac{P^A e}{P^B} = e \cdot \frac{\Omega_d^A}{\Omega_d^B} \cdot \frac{(1+\Psi_d^A)}{(1+\Psi_d^B)} = e \cdot \frac{\sum_{j=1}^n \left(\Omega_j^A \cdot d_j^A \cdot \sum_{i=1}^n \sigma_i^A m_{ij}^A\right)}{\sum_{j=1}^n \left(\Omega_j^B \cdot d_j^B \cdot \sum_{i=1}^n \sigma_i^B m_{ij}^B\right)} \frac{(1+\psi^A)}{(1+\psi^B)}, \tag{18}$$

where $\frac{\Omega_d^A}{\Omega_d^B}$ is the RTULC and $\frac{(1+\Psi_d^A)}{(1+\Psi_d^B)}$ is the RTPWR between commodity bundles d_d^A and d_d^B. The RER (18) is affected by the techno-distributive characteristics of all industries in A and B, just like in the ToT between any two commodities in (13). But (18) involves a qualitatively new feature affecting IRP: the semipositive quantities d_j^α can generate any positive value for Ψ_d^α, irrespective of the ψ_j^α, ψ^α, and m_{ij}^α.

Let us now complete the set of constraints that make relative prices to equal the RTULC involved in their production.

Proposition 4 *The real exchange rate between countries A and B exporting commodity baskets d^A and d^B equals the relative total unit labor costs involved in their production,*

$$\frac{P^A e}{P^B} = e \cdot \frac{\Omega_d^A}{\Omega_d^B}, \tag{19}$$

if and only if the relative total profits-wages ratios of associated with the production of commodities d^A and d^B in countries A and B, respectively, are the same, $\Psi_d^A = \Psi_d^B$.

For any given commodities baskets d^A and d^B, just like with Proposition 2, Proposition 4 requires the coordination of the technical-distributional characteristics between two economies. For the RER, we also have the situation where zero variability in the within countries total profits-wages ratios ($\sum_{i=1}^n \sigma_i^\alpha m_{ij}^\alpha = \sigma^\alpha$ for $j \in N$) cannot remove the influence of the between-countries distributive conditions:

$$\left.\frac{P^A e}{P^B}\right|_{\Psi_d^B = e\cdot\Psi^B}^{\Psi_d^A = \Psi^A} = e \cdot \frac{\Omega_d^A}{\Omega_d^B} \cdot \frac{\sigma^A}{\sigma^B} \cdot \frac{(1+\psi^A)}{(1+\psi^B)}, \tag{20}$$

In the extreme case of uniform *direct* ratios we have $\sigma^\alpha = 1$, so

Proposition 5 *Suppose uniform direct profits-wages ratios within countries, $\psi_j^A = \psi^A$ and $\psi_j^B = \psi^B$. Then,*

$$\left.\frac{P^A e}{P^B}\right|_{\psi_j^B = \psi^B}^{\psi_j^A = \psi^A} = e \cdot \frac{\Omega_d^A}{\Omega_d^B} \tag{21}$$

[15] The assumption of one price for domestic and foreign sales makes that the real exchange rate between country A and B coincide with their terms of trade.

if and only if the wage-shares in country A and B are equal, $\psi^A = \psi^B$.

Now, for any relative techno-distributive characteristics in countries A and B, we can have $\frac{P^A}{P^B} = \cdot\frac{\Omega_d^A}{\Omega_d^B}$ by selecting bundles d^A and d^B such that $\Psi_d^A = \Psi_d^B$:

Proposition 6 *There are commodity baskets $d^A = \{d_1^A, \ldots, d_n^A\}$ and $d^B = \{d_1^B, \ldots, d_n^B\}$ such that*

$$\frac{P^A e}{P^B} = e \cdot \frac{\Omega_d^A}{\Omega_d^B} \tag{22}$$

irrespective of the variability in their direct and the total profits-wages ratios.

International homogeneity of total profits-wages ratios is a sufficient but not necessary condition for Proposition 4 —the commodity bundles d^A and d^B are equally relevant in theory. But if Proposition 4 holds for any commodity basket, then $\Psi_d^A = \Psi_d^B$ can only hold under international uniformity in the total profits-wages ratios.

2.5 Long-period positions

The decomposition of national and international relative prices in (10), (13), and (18), as well as the conditions in Proposition 1 to 6, was an *accounting* exercise in market values for the economy depicted in Section 2.1. Once we introduce *economic* constraints on the total unit labor costs and total unit profits the relative prices $\frac{p_j}{p_k}$, $\frac{p_j^A}{p_k^B}$, and $\frac{P_d^A}{P_d^B}$ are no longer relative market prices and become theoretical relative prices.

A first set of constraints typically considered in theories of IRP-RC consists on long-period positions.[16] Some of these positions are (1) homogeneous labor L and a uniform wage rate w, (2) uniform profit rates $r_j = r$, and (3) production prices $\bar{p}_j$.

Under long-period positions, *national* relative market prices have as centers of gravity the relative production prices, which determine their long-run behavior, $\frac{p_j}{p_k} \approx \frac{\bar{p}_j}{\bar{p}_k}$. In addition, the total unit labor costs become $\Omega_j = wv_j$, where v_j is the total labor embodied in one unit of commodity j. Uniform profit rates across industries makes total profits per unit of output to be $\Pi_j = rK_j$, where K_j is the total capital advanced per

[16] These constraints, emerging from the process of competition between workers and between capitals in commodity producing economies under wage-labor, affect the nature of the labor, the rates of return of labor and capital advanced, and the characterization of prices. For thorough presentation on long-period positions see Garegnani (1976), Foley (2011, 2016), and Cogliano (2018, 2023). These theories typically embed the long-period positions within Sraffian linear production models. The latter are systems of equations where prices are determined under the assumptions of constant returns to scale or unchanging techniques of production and output scale and proportions.

unit of output valued at production prices. Hence, $\Psi_j = \frac{r}{w}\frac{K_j}{v_j}$ and the economy-wide direct profit-wage ratio ψ represent the rate of surplus value.

Under these constraints, the ToT (13) become

$$\frac{p_j^A}{p_k^B} \approx \frac{\bar{p}_j^A}{\bar{p}_k^B} = \frac{w^A v_j^A}{w^B v_k^B} \cdot \frac{1 + \frac{r^A}{w^A}\frac{K_j^A}{v_j^A}}{1 + \frac{r^B}{w^B}\frac{K_k^B}{v_k^B}}. \tag{23}$$

If we assume further that $r^\alpha = r$, then there will be international production prices. However, these constraints does not make $\frac{r}{w^A}\frac{K_j^A}{v_j^A} = \frac{r}{w^B}\frac{K_k^B}{v_k^B}$, i.e., $\frac{\bar{p}_j^A}{\bar{p}_k^B} = \frac{w^A v_j^A}{w^B v_k^B}$, so long-period IRP still depend on international technological and distributive conditions.

2.6 Dynamic counterpart of Propositions 1 to 5

The different relative prices (RP) reviewed in this section —i.e., (6), (13), (18), and (23)— have the same multiplicative structure: $RP = RTULC \cdot RTPWR$. This implies that $\widehat{RP} = \widehat{RTULC} + \widehat{RTPWR}$.[17] This implies that there will be a dynamic counterpart of propositions 1 to 5, under market or production prices. In each case, $\widehat{RP} = \widehat{RTULC}$ if and only if $\widehat{RTPWR} = 0$, that is, if and only if $\widehat{1+\Psi_d^\alpha} = \widehat{1+\Psi_d^\beta}$. In this case, $\frac{1+\Psi_d^\alpha}{1+\Psi_d^\beta}$ stay constant, even if $\Psi_d^\alpha \neq \Psi_d^\beta$, in contrast with propositions 1 to 5.[18]

3 The Theory of IRP based on 'Real Competition' and its Evaluation

3.1 The construction of the theory of IRP-RC

Shaikh argues that the *long-run* behavior of IRP is the outcome of international competition of capitals, just like competition of capitals at the national level determines the long-run NRP.[19] The competitive framework used to sustain this result is Shaikh's

[17] For any variable that is a continuous function of time $x = x(t)$, define $\dot{x} \equiv \frac{d}{dt}x$ and $\hat{x} \equiv \frac{\dot{x}}{x}$.

[18] The same reasoning applies for the expected value $\mathrm{E}\left(\widehat{RP}\right) = \mathrm{E}\left(\widehat{RTULC}\right) \iff \mathrm{E}\left(\widehat{RTPWR}\right)$, and the stationarity of the variable, $\frac{RP}{RTULC} \sim I(0) \iff RTPWR \sim I(0)$.

[19] 'This paper develops and tests a long run theory of the exchange rate based upon a classical approach to the theory of competition ... It is first applied to competition within one "nation" ... and then extended to the multi-currency case' (Shaikh, 1991, p. 2). See also (Shaikh, 1999, p. 12). Sarich (2006, pp. 470, 476-80) arrives to the same interpretation in providing an alternative explanation of the empirical evidence on the relationship between the RER and relative costs of production within the literature of the Harrod-Balassa-Samuelson effect.

own theory of 'Real Competition' (RC) —hence, IRP-RC.[20] The theory of RC results from adding more constraints to the long-period positions. Based on this framework, he develops a set of arguments to construct hypotheses attempting to yield the theory's main thesis: *for each pair of internationally* tradable *commodity bundles, the long-run behavior of their relative market prices is determined solely by their associated RTULC.*

3.1.1 Regulating capitals and their production prices

The theory of RC modifies the long-period positions by adding a second layer of determinations which avoids the homogeneity of conditions of production and reproduction within industries that was assumed in Section 2.1, but retain the competitive process and results for a subset of capitals, which he calls regulating capitals.[21]

Within any industry, different kind of factors produce a cost heterogeneity among capitals. Capitals within any industry will operate under different conditions of production. Only some of these conditions will be reproducible for newly incoming capitals. From these capitals producing under reproducible conditions, there will be a subset which operate at the lowest unit costs. These capitals are called the 'regulating capitals' of the industry and are able to set the price of the commodity which, under the law of one price, become the center of gravity of the rest of the prices fixed by the spectrum of capitals within this industry. The tendency towards price homogeneity coupled with heterogeneity in costs produces a spectrum of profit rates within this industry.

Now, in the search for the highest rate of return, competition between the regulating capitals across industries produces a tendency towards the equalization of their rates of profits. From this process it emerges a set of production prices for the regulating capitals, which constitute the new centers of gravity of market prices.

It is argued then that IRP are 'determined by the equalization of profit rates across international regulating capitals' (Shaikh, 1999, p. 0).[22] That is, the long-run behavior of international market relative prices are determined by the *relative production prices of the regulating capitals* in the industries producing these commodities, $p_j^A/p_k^B \approx \bar{p}_j^A/\bar{p}_k^B$, where $\bar{p}_j^\alpha$ now refer to production prices of the international regulating capitals.

[20] See Shaikh (1991, §I & II; 1995, pp. 70-1; 1999, pp. 1-3; 2002, pp. 6-7; 2016, §7.IV).

[21] See Shaikh (2016, §7.II & 7.IV).

[22] Most studies in the literature share this view. For instance, '[i]n the determination process of long-run real exchange rates [the] leading dynamic is the profit rate equalization' (Ersoy, 2010, p. 16).

3.1.2 The hypotheses constraining relative production prices of the regulating capitals

The thesis of theory of IRP-RC *requires one more layer of determinations*, this time constraining the production prices of the regulating capitals to have as a backbone or unique 'long-run determinant' the RTULC, ${p_j^A}/{p_k^B} \approx {\bar{p}_j^A}/{\bar{p}_k^B} \approx {\Omega_j^A}/{\Omega_k^B}$. It is argued that these constraints emerge from the forces of competition between regulating capitals.

Shaikh characterize this long-run behavior as a 'gravitation in the orbital sense' (1991, p. 1). Sometimes he means an oscillatory orbiting (1991, p. 7) —compatible with medium-term cycles (2016, p. 525)— or turbulent gravitation (2016, p. 530) of the IRP around the RTULC. Other times he treats $RTPWR = \frac{IRP}{RTULC}$ as a stationary process (2002, p. 9) with a mean-reversion (2016, p. 525) or, alternatively, he considers $({1+\Psi_j^A}/{1+\Psi_k^B})$ as a 'disturbance' term centered around one (e.g., Shaikh 2016, p. 518).

In order to obtain ${p_j^A}/{p_k^B} \approx {\Omega_j^A}/{\Omega_k^B}$, the theory of IRP-RC proceeds in two steps — one for the ToT and another for the RER. Firstly, it advances the *hypothesis* that the long-run behavior of NRP is determined by their RTULC, ${p_j}/{p_k} \approx {\Omega_j}/{\Omega_k}$. Then, it is *argued* that this constraint on NRP implies that the long-run ToT of any two tradable commodities is also determined by the RTULC associated in their production, ${p_j^A}/{p_k^B} \approx {\Omega_j^A}/{\Omega_k^B}$. Secondly, and building on the previous result, it advances the *hypothesis* that ${p_j^A}/{p_k^B} \approx {\Omega_j^A}/{\Omega_k^B}$ and then it is *argued* that this constraint on the ToT implies that the long-run RER of any two tradable commodity bundles is also determined by the RTULC associated in their production, ${P_d^A}/{P_d^B} \approx {\Omega_d^A}/{\Omega_d^B}$. Given the information in propositions 1 and 2, if we replace equalities $(=)$ for approximations $(\approx)$, then each of the hypotheses can be equivalently expressed as:

Hypothesis 1 $(\mathcal{H}_1)$ *The long-run behavior of relative prices of commodities j and k is determined by their relative total unit labor costs,*

$$\frac{p_j}{p_k} \approx \frac{\Omega_j}{\Omega_k} \quad \textit{for all} \quad j,k \in N, \tag{24}$$

or, the total profits-wages ratios of industries j and k are sufficiently close,

$$\Psi_j \approx \Psi_k \approx \Psi, \quad \textit{for all} \quad j,k \in N. \tag{25}$$

Hypothesis 2 $(\mathcal{H}_2)$ *The long-run behavior of the terms of trade of commodities j and k exported by countries A and B is determined by their relative total unit labor costs,*

$$\frac{p_j^A}{p_k^B} \approx \frac{\Omega_j^A}{\Omega_k^B} \quad \textit{for some} \quad j,k \in N, \tag{26}$$

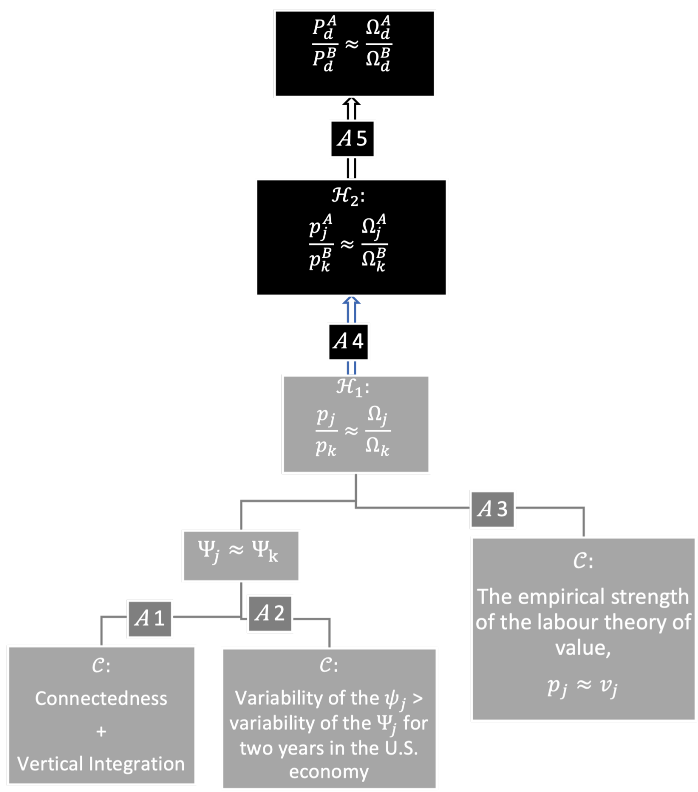

Figure 1: Structure of Shaikh's theory of international relative prices. $\mathcal{C}$, $\mathcal{A}$, and $\mathcal{H}$ refers to constraint, argument, and hypothesis, respectively.

or, the total profits-wages ratios of industry j in country A and industry k in country B are sufficiently close,

$$\Psi_j^A \approx \Psi_k^B, \quad \textit{for some} \quad j, k \in N. \tag{27}$$

From this structure, it is easy to see that the theory of IRP-RC relies on the theory of the long-run NRP. Overall, the logical strength of the thesis depends on the *soundness* in the construction of Hypothesis 1 and 2 and the *effectiveness* of these hypotheses to derive the results $p_j^A/p_k^B \approx \Omega_j^A/\Omega_k^B$ and $P_d^A/P_d^B \approx \Omega_d^A/\Omega_d^B$, respectively. Figure 1 provides a schematic representation of the theory by identifying (1) the hypotheses ($\mathcal{H}$), (2) the constraints supporting the hypotheses ($\mathcal{C}$), and (3) the argumentation linking the constraints/hypotheses to the desired results ($\mathcal{A}$). The dependence of the theory of IRP-TC to the theory of NRP is then represented with the gray boxes and lines. The top black boxes and lines link the hypotheses with the desired results on IRP.

3.2 The evaluation of the theory

We now provide an evaluation of the *soundness* of the hypotheses (Section 3.2.1), the *effectiveness* of the hypotheses (Section 3.2.2) and, finally, on the role played by the *forces of competition* in deriving the results that the long-run behavior of IRP is determined by their associated RTULV (Section 3.2.3).

3.2.1 The *soundness* of Hypothesis 1

The argument that Hypothesis 1 implies ${p_j^A}/{p_k^B} \approx {\Omega_j^A}/{\Omega_k^B}$ transfers some properties of *domestically* produced commodities to *internationally* produced commodities. It was originally stated in Shaikh (1991, p. 6; 1995, pp. 70-71) and it has been used as the foundation for empirical studies on the theory of IRP-RC ever since.[23] This hypothesis is sustained by three arguments on the existence of forces in the economy that make ${p_j}/{p_k} \approx {\Omega_j}/{\Omega_k}$. In Argument 1 and 2 these forces define explicit constraints on the Ψ_j in such a way that ${(1+\Psi_j)}/{(1+\Psi_k)}$ can be considered a 'disturbance' term close to one. In Argument 3 the forces manifest directly on the prices, such that ${p_j}/{p_k} \approx {\Omega_j}/{\Omega_k}$. Because Proposition 1 states that ${p_j}/{p_k} = {\Omega_j}/{\Omega_k} \iff \Psi_j = \Psi_k$, arguments 1-3 then reinforce each other. We now consider each one in detail and provide an evaluation.

Argument 1 ($\mathcal{A}_1$) *In a highly connected economy, the process of vertical integration makes the variability of the total profits-wages ratios* Ψ_j *to be sufficiently small, compared to whatever variability is in the direct profits-wages ratios* ψ_j*, so that* $\Psi_j \approx \Psi_k$ *for* $j, k \in N$*.*

Argument 1 was first presented in the literature on the classical theory of prices and the labor theory of value by Shaikh (1984, §4).[24] It was later introduced in the literature

[23] 'Within a nation, the relative prices of products can be well approximated by the relative total ... real unit labor costs ... In an international context, this same principle translates into the proposition that the relative common-currency prices of any two goods in the world market are regulated by the total real unit labor costs of the countries ...' (Antonopoulos, 1999, p. 58). See also Ge (1993, p. 266), Guerrero (1995, §4.3.A.a.1), Martínez-Hernández (2010, pp. 66-67), and Moreno-Rivas (2018, p. 25) provide similar arguments. See also the transition from NRP to IRP in Boundi-Chraki (2020, p. 2), Boundi-Chraki and Perrotini-Hernández (2021, p. 164), and Martínez-Hernández (2017, pp. 6-7), respectively. Fevereiro (2019), while considering the total profits-wages ratios between different countries (Ψ_j^A, Ψ_k^B), argues that $\Psi_j \approx \Psi_k$ implies $\Psi_j^A \approx \Psi_k^B$.

[24] And constantly reproduced in this literature: 'Now it is very important to recognise that the influence of [the $\frac{1+\Psi_j}{1+\Psi_k}$] is likely to be rather small, because its elements depend on the degree to which different convex combinations of direct profit–wage ratios differ from each other. As a consequence, even large variations in sectoral profit–wage rates are reduced to small ones in the corresponding

on the theory of IRP-RC by Shaikh (1991, 1995) and a great part of the literature had relied on these arguments ever since.[25] Its development is as follows: in every economy there is a given variability in the direct profits-wages ratios ψ_j. On the other hand, the total profits-wages ratios $\Psi_j = \sum_{i=1}^{n} \psi_j m_{ij}$ are weighted averages of these ψ_j. It is then argued that, in a highly connected economy, in the sense of a dense interindustry input-output network, the process of vertical integration makes each Ψ_j dependent on many ψ_j and with appropriate weights m_{ij} such that the variability of Ψ_j is sufficiently small in order for $\Psi_j \approx \Psi$ for $j \in N$, so that $^{p_j}\!/_{p_k} \approx {}^{\Omega_j}\!/_{\Omega_k}$.

This theoretical argument has two problems: The first one is the believe that the *expression* of prices in terms of vertically integrated industries adds a constraint that affects their properties. In Section 2.2 it was shown that prices in terms of direct and vertical integrated industries are *equivalent*. If it happens that $\Psi_j \approx \Psi_k$ this is not because of a strong or weak process of vertical integration. As shown in (4) and Appendix A.4, if $\Psi_j \approx \Psi_k$ it is because of particular statistical properties of the direct profits-wages ratios ψ_j and the weights m_{ij} as well as in their relation.[26]

The second problem in Argument 1 concerns the contribution of interindustry *connectedness* for $\Psi_j \approx \Psi_k$ for $j, k \in N$. General variations in industries' purchases of commodities produced by other industries do not affect Ψ_j systematically. Consider the Leontief-Sraffian models. It is well known that, for almost every configuration of the input-coefficient matrix, the matrix containing the information of the 'interindustry connectedness', $^{p_j}\!/_{p_k} = {}^{\Omega_j}\!/_{\Omega_k}$ or $\Psi_j = \Psi_k$ only by a fluke (see, Sraffa 1960, ch. III; Pasinetti 1977, pp. 82-4).[27] In fact, one of the factors that contributes to the low

integrated ratios. Therefore, equation (24) is a modified law of value, with [the $\frac{1+\Psi_j}{1+\Psi_k}$] containing some kind of probably negligible disturbance factors. If there is any transformation problem, it is most likely moderate. But this is an empirical question' (Fröhlich, 2012, p. 1112).

[25] For instance: 'For the reasons which have been explained by Shaikh, ... the second term can be viewed as a disturbance term so that than approximation to the above equation [$\frac{p_j}{p_k} \approx \frac{\Omega_j}{\Omega_k}$] is derived' Ge (1993, p. 259). See also (Guerrero, 1995, §4.3.A.a.1), Martínez-Hernández (2017, p. 5), and Boundi-Chraki and Perrotini-Hernández (2021, p. 164).

[26] In support of Argument 1, Shaikh (2016, pp. 387-8) argues that $\frac{1+\Psi_j}{1+\Psi_k} \approx 1$ even if Ψ_j differs considerably from Ψ_k. Shaikh construct an example where $\Psi_j = 0.40$ and $\Psi_k = 0.20$, that is, where Ψ_j doubles Ψ_k, and obtains $\frac{1+\Psi_j}{1+\Psi_k} = 1.16$, concluding that $\frac{p_j}{p_k}$ deviates from $\frac{\Omega_j}{\Omega_k}$ only by 16%. However, this result depends on the small value of Ψ_j and Ψ_k which makes $\frac{1+\Psi_j}{1+\Psi_k} \approx 1$, even if Ψ_j *triples* Ψ_k (for example $\frac{1+0.21}{1+0.07} \approx 1.13$). If instead Ψ_j doubles Ψ_k but with higher magnitudes, e.g., $\Psi_j = 2$ (VI profits doubles total wages) and $\Psi_k = 1$, then $\frac{1+\Psi_j}{1+\Psi_k} = \frac{3}{2} = 1.5$, so $\frac{p_j}{p_k}$ deviates from $\frac{\Omega_j}{\Omega_k}$ by 50%.

[27] Schefold (1976) and Bidard and Salvadori (1995) show that by marginally constraining the input matrices and the labor vectors (in order to produce what they call regular systems), price systems then have the property that for any n different profit rates there will be n linearly independent price vectors, so there is nothing that ensures that *in general* $^{p_j}\!/_{p_k} = {}^{\Omega_j}\!/_{\Omega_k}$ or $\Psi_j \approx \Psi_k$.

likelihood of a sufficiently low variability in the Ψ_j is precisely that industries use as means of production commodities produced in other industries. The only configuration of interindustry commodity flows which produces $\Psi_j = \Psi_k = \Psi$ is when the labor vector is proportional to the Perron-Frobenius eigenvector of the input matrix. That is, $\Psi_j = \Psi_k$ does not depend solely on interindustry connectedness.

Argument 2 ($\mathcal{A}_2$) *For the U.S economy, the standard deviation and the coefficient of variation of industries' total profits-wages ratios are considerably smaller than the same variability indicators for the direct profits-wages ratios, so that $\Psi_j \approx \Psi_k$ for $j, k \in N$.*

Shaikh (1984, p. 77; 2016, p. 388) compares the standard deviation and coefficient of variation of industries' direct and total profits-wages ratios and capital intensities for the U.S. economy for years 1947 and 1998 and reports that the variability of total ratios is considerably lower than that of the direct ratios, arguing with this that $\Psi_j \approx \Psi_k$ for $j, k \in N$ in this economy. Argument 2 is then empirical. The literature on the theory of IRP-RC has extrapolated this property of two years of the U.S. economy for their economies under study. Up to the authors best knowledge, this is the only available information on the statistical properties of profits-wages ratios or capital intensities.

The first problem of Argument 2 is that our critique of Argument 1 implies that the observed reduced variability of the Ψ_j *vis-a-vis* the ψ_j cannot be used as evidence in favor of sufficient $\Psi_j \approx \Psi_k$ for ${}^{p_j}\!/_{p_k} \approx {}^{\Omega_j}\!/_{\Omega_k}$: the variability of the Ψ_j do affect ${}^{p_j}\!/_{p_k}$ but the properties of latter do not depend on the reduced variability of the Ψ_j *compared* with the ψ_j. None of the authors of the literature have proven this proposition.

The second problem of Argument 2 is precisely the lack of knowledge of the statistical properties of profits-wages ratios. The literature on IRP-RC typically studies economies with deep technical and distributional differences (e.g., Spain-Germany). And yet, there is no empirical study of ψ_j, m_{ij}, and Ψ_j. Argument 2 cannot be considered a stylized fact supporting the constraint '$\Psi_j \approx \Psi_k$ for any market economy'.

Argument 3 ($\mathcal{A}_3$) *Due to the labor theory of value, market prices p_j are well approximated by the total quantities of labor v_j, so that $\frac{p_j}{p_k} \approx {}^{\Omega_j}\!/_{\Omega_k}$.*

Part of the literature on the theory of IRP-RC relies (completely or complementary) on a version of the Ricardian or Marxian theory of value[28] which argue that the centers

[28] 'If we apply Ricardo's theory of price determination, as developed by Pasinetti ... we cannot assume that the profit rate is zero; neither are we estimating natural prices with a positive uniform

of gravity of relative market prices are given by their relative labor-values, that is, the relative quantities of total labor contained in the production of the commodities, such that ${p_j}/{p_k} \propto \frac{v_j}{v_k} = {\Omega_j}/{\Omega_k}$.[29] This view is also adopted in studies of international trade and unequal exchange based on the theory of RC (e.g., Seretis and Tsaliki 2016, pp. 444-5; Tsaliki et al. 2017, pp. 1052-5). Argument 3 differs from Argument 1 and 2 in that the former does not rest on an explicit constraint on the Ψ_j, but it rather rests either on the axiom ${p_j}/{p_k} \approx {v_j}/{v_k}$ from the operation of the Ricardian or Marxian 'law of value'[30] or as a hypothesis sustained by empirical evidence on the relationship between market prices, direct prices, and production prices.[31]

The literature review by Basu (2017, §4) organizes these empirical exercises in two classes: those that conduct the regression $\ln \frac{p_j}{p} = \alpha + \beta \ln \frac{v_j}{v}$ and those that evaluate the proximity of vectors $(\frac{p_1}{v_1}, \frac{p_2}{v_2}, \ldots, \frac{p_n}{v_n})$ and $(1, 1, \ldots, 1)$ using different distance measures, where p and v are references prices and quantities of labor.[32] One problem with this approach is that there is no evaluation of the pairs of relative market and direct prices $({p_j}/{p_k}, {v_j}/{v_k})$ for $j, k \in N$, which would involve the study of $\frac{n(n-1)}{2}$ relative prices (Fröhlich, 2012, p. 1110). In studying only the relationship between p_j and v_j it is sustained that ${p_j}/{p_k} \approx {\Omega_j}/{\Omega_k}$ for $j, k \in N$.

profit rate [and] uniform capital–labour ratios across sectors ... we are neither estimating natural prices between countries as if they were regions within one country ... The proposition is that ... we are calculating vertically integrated unit labour costs (VIULC). But according to Ricardo, these average VIULC will determine average prices in each economy and, therefore, the ratio of the two countries' VIULC determines the real exchange rate between their currencies' (Ruiz-Nápoles, 2004, p. 76). See also Ersoy (2010, p. 16), Martínez-Hernández (2017, p. 6), Féliz and Pedrazzi (2019, p. 60), Boundi-Chraki (2019, pp. 125), and Poulakis and Tsaliki (2023a, p. 608-9; 2023b, p. 9). Aglietta and Oudiz (1984, p. 94) *define* the RER as the relative unit labor costs based on the Ricardian theory of value.

[29] For the Ricardian version the substance of labor-values is 'labor time' whereas for the Marxian version 'socially necessary labor time'. In their mathematical modeling, the v_j are re-scaled such that the sum of prices corresponds to the sum of values. For an alternative understanding of the labor theory of value see Foley (1986, ch. 2; 2011; 2016).

[30] e.g., Ruiz Nápoles (2004, pp. 76-7; 2010, pp. 18-21), Martínez-Hernández (2010, pp. 65-6), and Boundi-Chraki (2019, pp. 124-5).

[31] Mariolis and Tsoulfidis (2016, chap. 3) and Shaikh (2016, chap. 8, sect. V, VI, VIII, IX) provide a thorough exposition of the methodology. Cheng and Li (2019) provides a survey of the literature and Basu (2017) suggests alternative methodological procedures which prevent potential problems. Ochoa (1984) conducts the most detailed study and Işıkara and Mokre (2021) the most extensive one.

[32] The average deviations between individual market and direct prices is typically small across different economies and through the use of different models and metrics —typically less than 20%. The results are robust for (1) a wide sample of countries with different development levels, (2) different scalar metrics to measure the deviations (e.g., mean absolute deviations, mean-weighted absolute deviations, among others) (see Mariolis and Tsoulfidis 2016, ch. 4; Shaikh 2016, ch. 9, §IV), and (3) different Leontief-Sraffian models (with and without fixed capital and turnover rates, and with ex-post or ex-ante wages).

3.2.2 The *effectiveness* of the Hypothesis 1 and 2 deriving the results $p^A_j/p^B_k \approx \Omega^A_j/\Omega^B_k$ and $P^A_d/P^B_d \approx \Omega^A_d/\Omega^B_d$

The literature on the theory of IRP-RC sustains that Hypothesis 1, and therefore Argument 1 to 3, *imply*, with no further constraints, that the long-run behavior of the ToT (Argument 4) and RER (Argument 5) is governed by the RTULC involved in the production of the tradable commodity bundles composing the IRP.

Argument 4 ($\mathcal{A}_4$) *The determination of the long-run behavior of NRP by their relative RTULC implies that the long-run behavior of the terms of trade between two commodities exported by two countries is determined by their respective RTULC.*

Argument 4 transfers some properties of NRP between any two commodities to the ToT of any two commodities but produced in different countries. Given Proposition 1 and 2, Argument 4 can be stated as $p_j/p_k \approx \Omega_j/\Omega_k \rightarrow p^A_j/p^B_k \approx \frac{\Omega^A_j}{\Omega^B_k}$ or $\Psi^\alpha_j \approx \Psi^\alpha_k \approx \Psi^\alpha \rightarrow \Psi^A_j \approx \Psi^B_k$. If the approximations are replaced with equal signs, then Argument 4 can be stated as Proposition 1 implies Proposition 2.

It is evident then that Argument 4 has an efficiency problem: $\Psi^A_j = \Psi^A$ and $\Psi^B_j = \Psi^B$ does not imply that $\Psi^A = \Psi^B$. The problem does not originate in the degree of heterogeneity of the profits-wages ratios *within* countries. The only configuration of these ratios which supports Argument 4 is $\Psi^A_j \approx \Psi^B_k \approx \Psi$. Hence, if $p^A_j/p^A_k \approx \Omega^A_j/\Omega^B_k$ is meant for every tradable commodity, then $\Psi^A_j \approx \Psi^B_k$ for $j, k \in N$ implies *international uniformity* in total profits-wages ratios. The dependence of IRP to international techno-distributive differences has escaped the attention from the literature.[33] One side effect of this omission is that there is a vacuum of knowledge within the literature on the statistical properties of *international* profits-wages ratios.[34]

The assumption of international homogeneity of total profits-wages ratios across industries and countries contradicts one of the plead pillars of the theory of IRP-RC, namely, the none trivial techno-distributive international differences:[35]

[33] An expression similar to (13), where all components from IRP are explicitly stated before constraints are introduced, only appeared in the literature until Góchez-Sevilla and Tablas (2013, p. 31) and Shaikh (2016, p. 518). Before that, it has been assumed that NRP close to their RTULC must imply the same faith for IRP. For instance, '[i]f we let p denote unit price, and v denote the unit vertically integrated labor cost of the regulating producer, then for any two industries within a nation we may write ... $(p_i/p_j) \approx (v_i/v_j)$ ... The same principle may be applied on an international scale, modified only to take into account the distinction between national currencies' (Shaikh and Antonopoulos, 2012, p. 209). See also Poulakis and Tsaliki (2023a, p. 608-9; 2023b, p. 9-10).

[34] Fevereiro (2019, §3) is the only exception —with his study of national wage-shares.

[35] See also Shaikh (1980, §II.2, specially footnote 27).

> [I]t is an essential feature of our analysis that production conditions and real wages are assumed to differ across countries. Neoclassical theory tends to assume that production functions are similar across countries ... Yet nothing could be further from the empirical facts ... Dollar, Wolff, and Baumol (1988) find that for any given industry, productivity varies substantially across countries ... such international productivity variations are largely explained by corresponding variations in real capital-labor ratios ... countries with higher productivity and/or capital-labor ratios in one industry tend to have higher measures in all industries ... and countries with higher productivity in a given industry tend to also have higher wages in the same industry ... On this basis we assume that real wages and technology are determined locally in each nation (Shaikh, 1995, pp. 68-9).

Authors from the literature argue on the existence of these international differences in technology and distribution.[36] Given these alleged international differences, it is hard to sustain $\Psi_j^A \approx \Psi_k^B \approx \Psi$ for $j, k \in N$. Even if we remove the international differences in the techniques of production, say by assuming long-period positions and uniform direct capital intensities (which implies uniformity in the direct profits-wages ratios), Proposition 3 shows that the ToT still depends on the relative wage-shares or rates of exploitation of countries A and B, $\frac{p_j^A}{p_k^B} = \frac{\Omega_j^A}{\Omega_k^B} \cdot \frac{1+\psi^A}{1+\psi^B}$.[37]

Argument 5 ($\mathcal{A}_5$) *Because what happens to individual relative prices happens for relative price indices, then* $\frac{p_j^A}{p_k^B} \approx \frac{\Omega_j^A}{\Omega_k^B}$ *implies that* $\frac{P_d^A}{P_d^B} \approx \frac{\Omega_d^A}{\Omega_d^B}$.

The study of the RER introduces commodity baskets d^A and d^B for the construction of price indices P_d^A and P_d^B. Argument 5 can be stated, if approximations are substituted for equalities, as Proposition 2 implies Proposition 4,[38] that is, $\Psi_j^A \approx \Psi_k^B \rightarrow \Psi_d^A \approx \Psi_d^B$.

This argument holds only in highly restrictive cases. On the one hand, for a given commodity baskets, the general validity of Argument 5 requires the assumption of

[36] 'With regard to international competition ... one would expect that techniques of production of any World Industry, where individual firms are spread out through various countries, will vary from one nation to another as well' (Martínez-Hernández, 2010, p. 64). See also Ruiz-Nápoles (2004, p. 73), Góchez-Sevilla and Tablas (2013, p. 7), and Boundi-Chraki (2019, p. 120).

[37] The research program of Martínez-González and Valle-Baeza has documented the international differences in rates of surplus value. See inter alia, Martínez-González (2005), Martínez-González and Valle-Baeza (2023), and Martínez-González et al. (2019).

[38] 'In keeping with the empirical results of chapter 9 (see equation 9.5), we can also assume that the disturbance term is relatively small so that $[1 + \Psi_d^A / 1 + \Psi_d^B \approx 1]$' (Shaikh, 2016, p. 518). Some studies even assume that $RER_{B_d}^{A,d} \approx RTULC_{B_d}^{A,d}$ derives from the approximate verification of the conditions of Proposition 1 (e.g., Boundi-Chraki and Perrotini-Hernández 2021, p. 164).

international uniformity in the total profits wages ratios, $\Psi_j^A \approx \Psi_j^B \approx \Psi \geqslant 0$ for $j \in N$. On the other hand, Proposition 6 shows that it is always possible to find commodity baskets which brings uniformity to the index of total profits-wages ratios f*or any set of total profits-wages ratios* involved in the construction of the price indices. This 'quantity-side' of IRP determination has been absent in the literature.[39]

3.2.3 On the role of competition constraining production prices

We close the evaluation of the theory of IRC-RC by considering the role played by the forces of competition given by either the long-period positions or the constraints from the theory of RC. The process of mobility of all capitals or of the regulating capitals seeking the highest rates of return and producing a tendency towards the equalization of their rates of profit cannot produce the equality of the total profits-wages ratios of any two commodity baskets. Under long-period positions, the ratios $\Psi_j = \frac{r}{w}\frac{K_j}{v_j}$ depend on distributional and technical dimensions of the economy. The literature has not shown how capital mobility modifies $\frac{K_j}{v_j}$ such that $\Psi_j = \Psi_k$ for $j \in N$. Actually, the theoretical developments on the formation of an average profit rate and production prices usually take as given the technique of production. Another process of competition, i.e., technical change, could definitely constrain the ratios Ψ_j by affecting the capital intensities $\frac{K_j}{v_j}$. However, technical change at the capital level is more likely to increase rather than reduce the variability of the profit rates. Arguments 1-3 from the theory of IRP-RC do not depend on the emergence of production prices.[40]

[39] To see the problems of Argument 5, let us consider a simple example. Suppose that $N = \{1, 2, 3, 4\}$, $d^A = \{d_1^A, d_2^A, 0, 0\}$ and $d^B = \{0, 0, d_3^B, d_4^B\}$, that is, A exports commodities 1 and 2 to B and B exports commodities 3 an 4 to A. Because $\frac{P^A_{\{1,2\}}}{P^B_{\{3,4\}}} = \frac{p_1^A d_1^A + p_2^A d_2^A}{p_3^B d_3^B + p_4^B d_4^B} = \frac{(\Omega_1^A d_1^A + \Pi_1^A d_1^A) + (\Omega_2^A d_2^A + \Pi_2^A d_2^A)}{(\Omega_3^B d_3^A + \Pi_3^B d_3^A) + (\Omega_4^B d_4^B + \Pi_4^B d_4^B)} = \frac{\Omega_d^A}{\Omega_d^B} \cdot \frac{1 + \left(\delta_1^A \Psi_1^A + \delta_2^A \Psi_2^A\right)}{1 + \left(\delta_3^B \Psi_3^B + \delta_4^B \Psi_4^B\right)}$, the RER is then $\frac{P^A_{\{1,2\}}}{P^B_{\{3,4\}}} = \frac{\Omega_d^A}{\Omega_d^B} \cdot \frac{1+\Psi_d^A}{1+\Psi_d^B}$, where $\Omega_d^A = \Omega_1^A d_1^A + \Omega_2^A d_2^A$, $\Omega_d^B = \Omega_3^B d_3^B + \Omega_4^B d_4^B$, $\Psi_d^A \equiv \delta_1^A \Psi_1^A + \delta_2^A \Psi_2^A$, $\Psi_d^B \equiv \delta_3^B \Psi_3^B + \delta_4^B \Psi_4^B$, $\delta_j^A \equiv \frac{\Omega_j^A d_j^A}{\Omega_1^A d_1^A + \Omega_2^A d_2^A}$ and $\delta_k^B \equiv \frac{\Omega_k^B d_k^B}{\Omega_3^B d_3^B + \Omega_4^B d_4^B}$, and $\delta_1^A + \delta_2^A = \delta_3^B + \delta_4^B = 1$. Suppose in addition that Hypothesis 1 holds for the trade between commodities 1 and 3 and 2 and 4, i.e., that $p_1^A/p_3^B \approx \Omega_1^A/\Omega_3^B$ and $p_2^A/p_4^B \approx \Omega_2^A/\Omega_4^B$, so $\Psi_1^A \approx \Psi_3^B$ and $\Psi_2^A \approx \Psi_4^B$. From the equation for $P^A_{\{1,2\}}/P^B_{\{3,4\}}$, we can see that in general is not possible to have $P^A_{\{1,2\}}/P^B_{\{3,4\}} \approx \Omega_d^A/\Omega_d^B$ because we cannot say that *in general* $\delta_1^A \Psi_1^A + \delta_2^A \Psi_2^A \approx \delta_3^B \Psi_1^A + \delta_4^B \Psi_2^A$. We would need the special constraints of international uniformity in the ratios $\Psi_1^A \approx \Psi_2^A \approx \Psi_3^B \approx \Psi_4^B \approx \Psi$ or specific commodity baskets d^A and d^B such that the weights $(\delta_1^A, \delta_2^A, \delta_3^B, \delta_4^B)$ make $\delta_1^A \Psi_1^A + \delta_2^A \Psi_2^A \approx \delta_3^B \Psi_3^B + \delta_4^B \Psi_4^B$.

[40] The presentation in sections 2.1-2.4 of necessary and sufficient conditions for IRP to equal their respective RTULC was conducted on the basis of market prices and from a cost-of-production perspective. So constraints $\Psi_j = \Psi_k$, $\Psi_j^A = \Psi_k^B$, or $\Psi_d^A = \Psi_d^B$ could in principle be consistent with alternative theories of competition, as has been sometimes suggested in the literature —e.g., '[t]his result can be derived either from a classical framework, along the lines of Ricardo, Sraffa, and Pasinetti, or from a mark-up pricing framework as in Eichner' (Shaikh, 1995, p. 71). Antonopoulos (1999, p. 58) shares

4 Stylized Facts on Profits-Wages Ratios and Measuring the Likelihood of $\Psi_j^A \approx \Psi_k^B$

4.1 Methodology and database construction

In the previous section we showed that the main thesis of the theory of IRP-RC, i.e., that the long-run behavior of any pair of tradable commodities is determined exclusively by its RTULC, $\frac{p_j^A}{p_k^B} \approx \frac{\Omega_j^A}{\Omega_k^B}$, is equivalent to the hypothesis of world uniformity in TPWR, i.e., $\Psi_j^\alpha \approx \Psi$ for $j \in N$ and $\alpha = A, B$. The reconstruction and evaluation of the theory showed that (i) this hypothesis has not been identified by the literature, (ii) there is a vacuum of knowledge on the statistical properties of the Ψ_j^α, and, therefore, (iii) there is no empirical evidence on the likelihood of $\Psi_j^A \approx \Psi_k^B$ nor of the inter-country uniformity in TPWR, $\Psi_j^A \approx \Psi^A$. The same applies to the hypothesis of similar wage shares $(1+\psi^A)^{-1} \approx (1+\psi^B)^{-1}$, which is a necessary constraint for the theory of IRP-RC under the hypothesis of within-country homogeneity in TPWR.

Based on this lacuna of empirical information, in this section we shall conduct the first large-scale empirical study of the ratios Ψ_j, $(1+\psi)^{-1}$, and $\frac{K_j}{v_j}$ and report (i) stylized facts on these ratios (Section 4.2) and (ii) empirical evidence on the hypothesis of $\Psi_j^A \approx \Psi_k^B$ (Section 4.3). For the former, we shall employ empirical density functions to summarize the statistical information on the ratios. As for the latter, we shall exploit the information obtained from the posterior distribution of the mean parameters of all countries' Ψ_j to perform a hypothesis test of $\Psi_j^A \approx \Psi_k^B$, i.e., the Bayesian estimation and comparisons of the posterior distribution of the mean of Ψ_j^α.

Appendix B provides the details on the database construction. Here we just provide some details. For the study of the TPWR (ψ_j, Ψ_j) and total capital intensities $\frac{K_j}{v_j}$ we draw from the World Input-Output Database (WIOD) (Timmer et al., 2016). This database contains information on national input-output tables and industry-level output and input information at market prices and for a period of 15 years (2000-2014) and 42 countries, $\alpha = \mathrm{AUS}, \mathrm{AUT}, \ldots, \mathrm{USA}$. This sample contains high- and middle-income countries which account for 86% of the world economy in 2016. We eliminated the industries for which there is no information, leaving approximately between 33-54 industries for each country-year, $j = 1, 2, \ldots, 33$ or 54. Table B.1 contains the list of

this view. For instance, because $(1+\Psi_j)$ can be interpreted as a mark-up applied to the total unit labor costs $p_j = (1+\Psi_j)\Omega_j$, then $\Psi_j^A \approx \Psi_k^B$ refers to a process of equalization of the degrees of monopoly in the different industries.

countries and their number of industries with nonzero information. Table B.2 describes each industry. The computation of the Ψ_j, K_j, and v_j are based on standard input-output models relying on Leontief's inverse. The study of the wage-shares $(1+\psi)^{-1}$ will be based on the Penn World Tables (Feenstra et al., 2015).

4.2 Profits-wages ratios across countries and industries

Figure 2 presents the first batch of evidence in the form of the empirical distributions of industries' TPWR within a country, $\Psi_1^\alpha, \ldots, \Psi_n^\alpha$. The first point to stand out is the high degree of organization of the data across economies. The empirical densities display high homogeneity in their functional forms characterized by strong clustering of the data around a single mode with limited variability characterized by slight skewness towards lower values and a fat right-hand tail. These high-valued observations sometimes produce a slight bimodality, such as in Australia (AUS), Spain (ESP), and India (IND). In advanced countries, the empirical densities seem time invariant while for countries undergoing a strong process of structural change, such as China (CHN) or Romania (ROU), we observe a shift in location.[41]

Remarkably, this characterization of the empirical distributions is also present in industries' TPWR *across* countries, $\Psi_j^{\text{AUS}}, \ldots, \Psi_j^{\text{USA}}$ —see Figure 3. Visual inspection shows that the statistical organization of the data even improves, showing more homogeneity and time-invariance.[42] Distributions are smooth and unimodal, with a constrained shape featuring a fat right-hand tail and a limited number of outliers. Locational shifts are modest and irregularities are reduced considerably.

Under long-period positions in labor and the assumption that wages are not part of the capital advanced, we can decompose the TPWR into a distributional and a technological component, $\Psi_j^\alpha = \frac{r_j^\alpha}{w^\alpha}\frac{K_j^\alpha}{v_j^\alpha}$. Are the statistical regularities in the Ψ_j^α also present in the capital intensities $\frac{K_j^\alpha}{v_j^\alpha}$? If we also assume $r_j^\alpha = r^\alpha$, then $\Psi_j^\alpha = \frac{r^\alpha}{w^\alpha}\frac{K_j^\alpha}{v_j^\alpha}$ and changes in the ratio $\frac{r^\alpha}{w^\alpha}$ should only affect the location of the within-country distribution of Ψ_j^α, leaving the rest of its statistical properties to the distribution of the $\frac{K_j^\alpha}{v_j^\alpha}$. We find evidence for this. Figure C.1 in Appendix C reports the empirical distribution of the total capital intensities $\frac{K_1^\alpha}{v_1^\alpha}, \ldots, \frac{K_n^\alpha}{v_n^\alpha}$. Visual inspection suffices to detect the strong

[41] There are some irregular cases characterized by (i) dual-peaked distributions, as in some years for Austria (AUT) and for all years in South Korea (KOR), and (ii) less stable functional forms, such as in Cyprus (CYP), Luxembourg (LUX), and Russia (RUS).

[42] Each density is estimated from approximately 42 countries. The information contained in figures 2 and 3 is exactly the same —the former is organized by country and in the latter by industry.

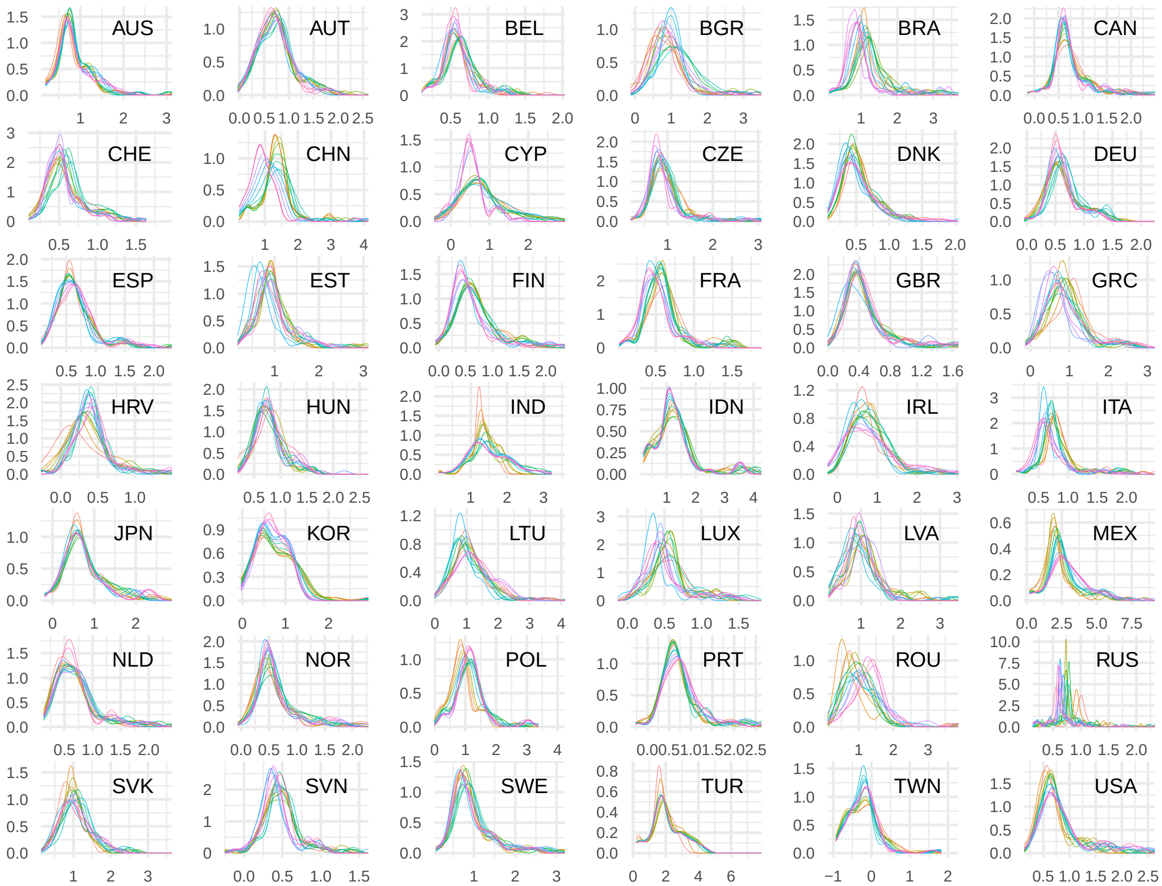

Figure 2: Total profits-wages ratios of industries within countries, 2000-2014. *Source*: authors' calculations using the World Input-Output Database.

similarity in the empirical densities of Ψ_j^α and $\frac{K_j^\alpha}{v_j^\alpha}$: the distributions are well-behaved, clustering strongly around the mode with limited variability and high symmetry around the mode.[43] Hence, the statistical regularities in $\frac{K_j^\alpha}{v_j^\alpha}$ seem relevant for Ψ_j^α.[44]

Finally, let us look at the empirical densities of the wages shares in Figure 4. The data shows that $(1+\psi^\alpha)^{-1}$ shares basically the same statistical properties with Ψ_j^α and $\frac{K_j^\alpha}{v_j^\alpha}$. The right panel, drawing from the WIOD, shows that the wage share of 42 high- and middle-income countries display a highly peaked unimodal empirical density, with some changes in location and scale during the period 2000-2014. The left panel, containing data for 43 countries from a wider spectrum of income levels and for the

[43] The only exception to this being the strong bimodality of India (IND) and the slight one in South Korea (KOR). The data from other countries, such as Russia (RUS), Greece (GRC), or China (CHN), is less organized, with strong variability in the location and the functional form.

[44] If we also consider the the empirical densities of the cross country total capital intensities, $\frac{K_j^{\mathrm{AUS}}}{v_j^{\mathrm{AUS}}}, \frac{K_j^{\mathrm{AUT}}}{v_j^{\mathrm{AUT}}}, \ldots, \frac{K_j^{\mathrm{USA}}}{v_j^{\mathrm{USA}}}$, see Figure C.2 in Appendix C, we also observe similarities with the characteristics of the empirical densities in Figure 2, 3, and C.1.

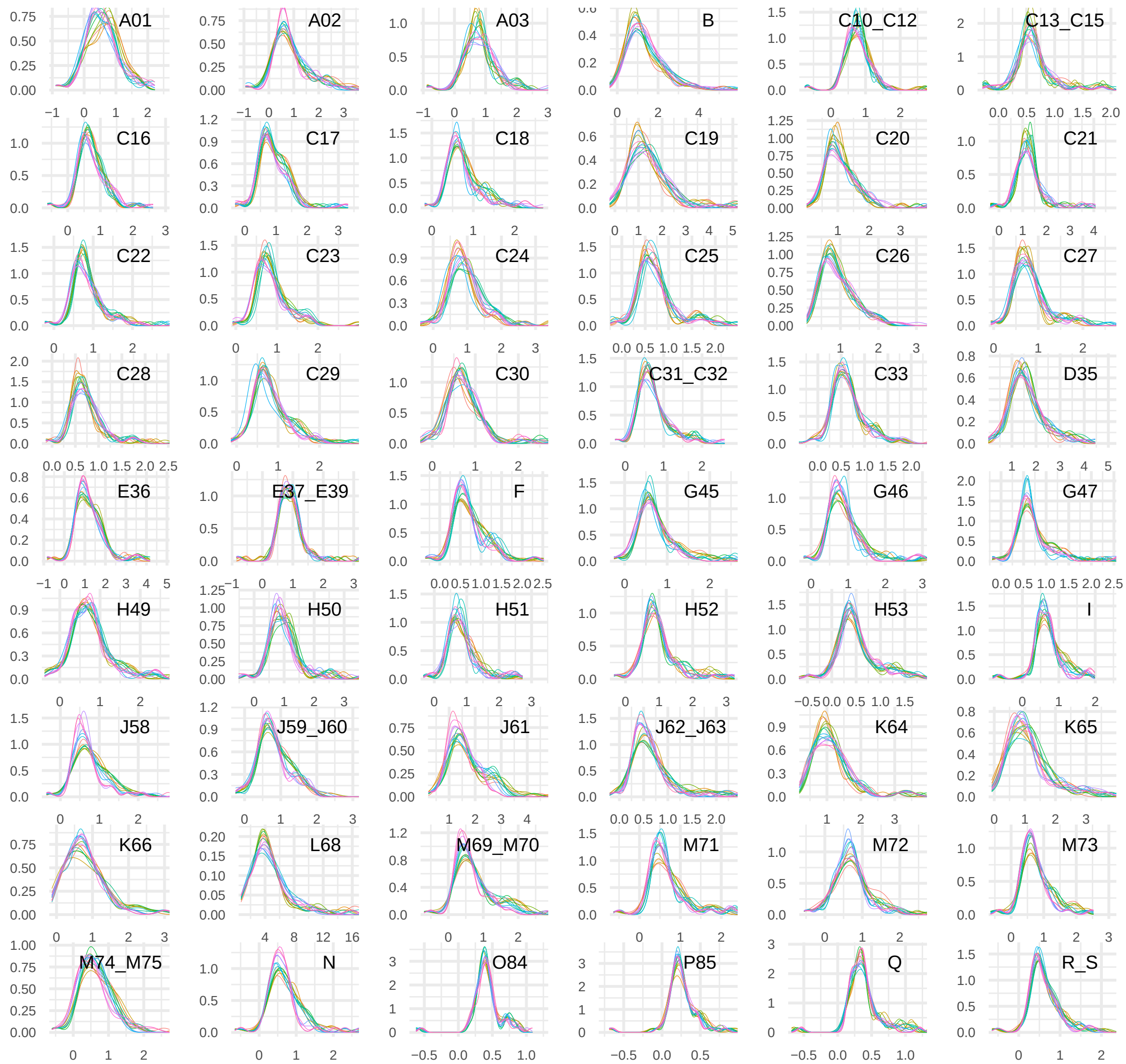

Figure 3: Total profits-wages ratios of industries across countries, 2000-2014. *Source*: authors' calculations using the World Input-Output Database.

period 1970-2014, tells a similar story but with a process of structural change showing a lower average wage shares and a transition from bi-modality to uni-modality.

All the variates Ψ_j^α, $\frac{K_j^\alpha}{v_j^\alpha}$, and $(1+\psi^\alpha)^{-1}$ show a considerable statistical organization, with persistent and generalized features like strong concentration around a unique mode and limited variability and asymmetry. This implies the existence of technological and distributive constraints operating within countries-across industries and within industries-across countries —otherwise we would be observing close-to-uniform empirical distribution across a range of relevant values. If it is true that there is limited variability and concentration around a central value, there are also differences in loca-

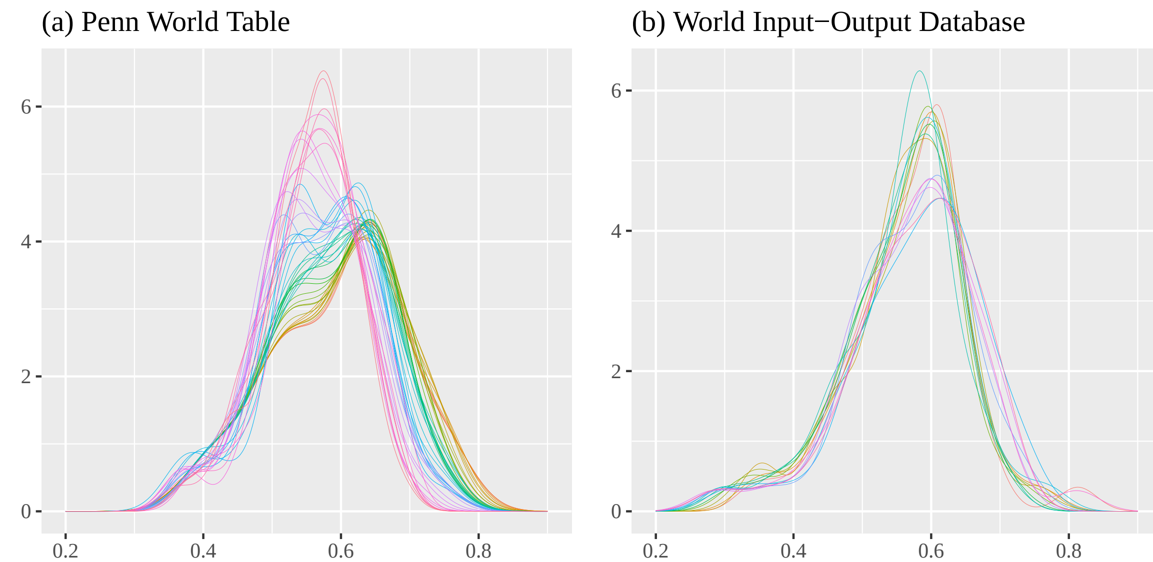

Figure 4: Wage-shares across countries: 1970-2014 for the PWT v10 and 2000-2014 for the WIOD release 2016. *Note*: Warm colors indicate older dates. *Source*: authors' calculations.

tion and scale. Do these statistical characteristics support the hypothesis of $\Psi_j^A \approx \Psi_k^B$? The remaining variability of Ψ_j^α and the variability in their central tendencies might prevent the sufficiently close proximity in the TPWR required for the long-run behavior of the IRP of any pair of commodity bundles to be determined by their respective RTULC. The next section provides empirical results for these questions.

4.3 Testing the Hypothesis of Ψ_j^A and Ψ_k^B mean-overlap

While the statistical variability of the Ψ_j^α within countries calls into question the assumption that $\Psi_j^\alpha \approx \Psi^\alpha$, it is not enough to reject the assumption that $\Psi_j^A \approx \Psi_k^B$ —The variability within countries could potentially cancel out. Given the unimodality and limited variability of the empirical distributions of the Ψ_j^α, their central tendencies, like their mean, constitute a representative Ψ_j^α. Can we say that the central tendencies of the Ψ_j^α distributions overlap, such that $\frac{1+\Psi_j^A}{1+\Psi_k^B} \approx 1$? The likelihood of the mean overlap is one approach to evaluate the hypothesis that $\Psi_j^A \approx \Psi_k^B$.

To test the empirical strength of the hypothesis, we rely on a Bayesian approach to derive the full pair-wise distribution of credible values of the mean TPWR among all countries —See Appendix D.1 for details on the methodology. Our approach draws from Kruschke (2013), Gelman et al. (2013), and Vehtari et al. (2017) to define a hypothesis

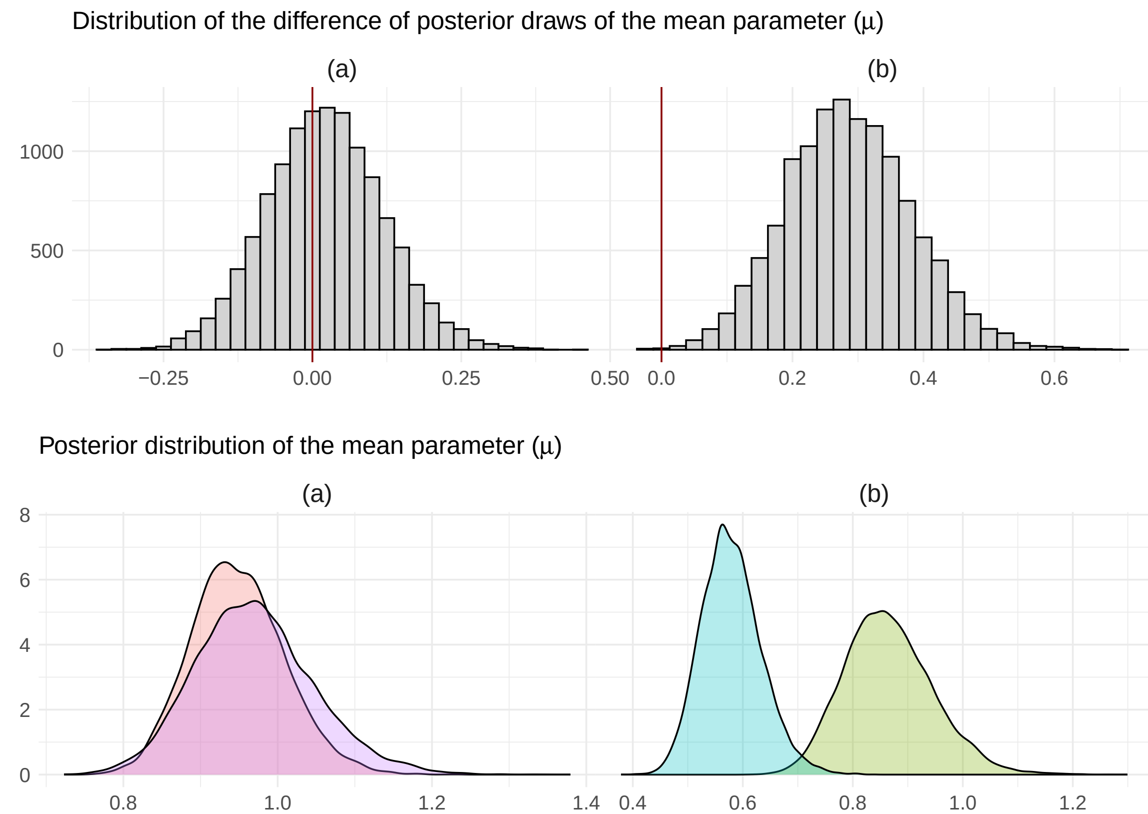

Figure 5: Illustration of the relationship between posterior distribution of the mean parameter μ_{Ψ^α} of the total profits-wage ratios Ψ_j^α and the credibility intervals of their difference. *Source*: authors' calculations.

test based on the posterior distribution of the mean of Ψ_j^α. In particular, we want to know if the means of the distributions in Figure 2 are credibly different from each other, in which case we would reject the null hypothesis that $\Psi_j^A \approx \Psi_k^B$. In this case, zero will fall outside the 95% highest density interval of the distribution of differences. To take the credible combination of means *and* standard deviations into account, we compute the effect sizes of the difference of means —See Appendix D.2.

Figure 5 illustrates this approach by displaying two rows of plots. The first one shows the distribution of the difference of the random draws from two hypothetical countries' posterior distribution of the mean parameter ($\mu_{\Psi^A} - \mu_{\Psi^B}$). The second row shows the two posterior distributions from which the difference is computed ($\mu_{\Psi^A}, \mu_{\Psi^B}$). Whenever the two posterior distributions overlap, as in the plots *(a)* on the left, the mode of the resulting distribution of the differences will be located at zero. Conversely, as in the right-hand plots *(b)*, almost the entire range of credible values is distinct from zero, indicating that the likelihood of the two original Ψ_k^α frequencies to be close enough

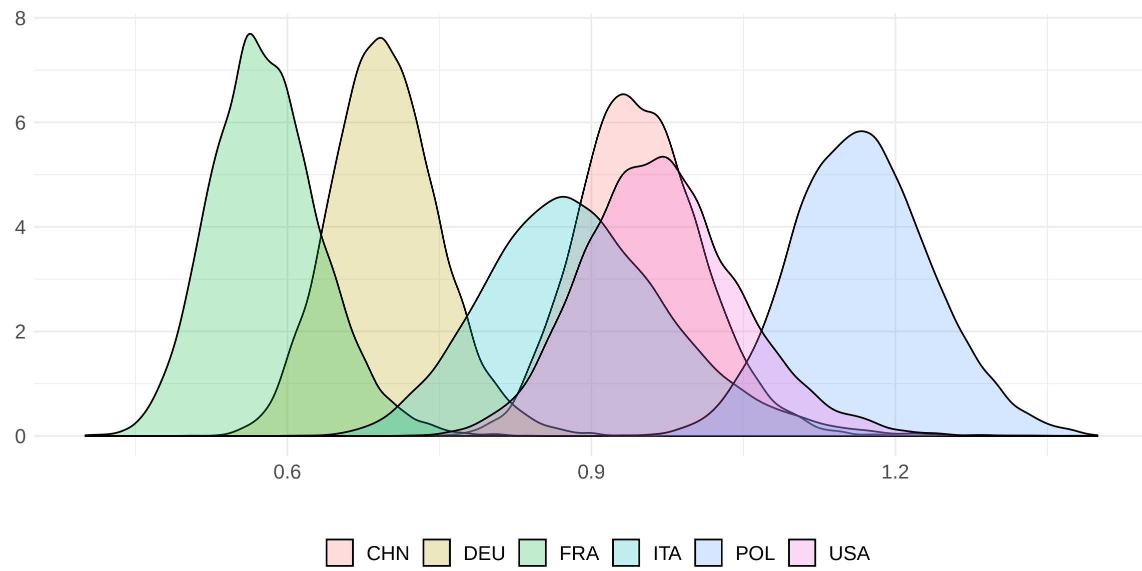

Figure 6: Posterior distribution of the mean parameter μ_{Ψ^α} of the total profits-wage ratios Ψ_j^α of a selection of countries in 2014. *Source*: authors' calculations.

is fairly low.[45] We can apply this technique to evaluate the hypothesis that $\Psi_j^A \approx \Psi_k^B$ obtaining the probability that *on average* the two distributions are sufficiently close.

To derive estimates of the means of the TPWR (μ_{Ψ^α}) we need to choose the distribution family that best represents the data-generating process that is most likely to be behind each Ψ_j^α. Based on visual inspection of the empirical distributions of Ψ_j^α and model evaluation using leave-one-out cross-validation (Vehtari et al., 2017), we choose the Gamma distribution as a reasonable approximation to the Ψ_j^α —See Appendix D.3.

Figure 6 reports the posterior distributions of the mean of the Gamma model μ_{Ψ^α} for six countries in 2014: China (CHN), Germany (DEU), France (FRA), Italy (ITA), Poland (POL), and the United States (USA). The country selection illustrates the variability of credible values. While the USA and China are closely aligned, Poland and Germany do not share a single point in common. On the other hand, France and Germany are closer, but the probability that the mean of their distributions overlap is relatively low. Based on the distribution of the most probable values of the mean, we can be confident about the high likelihood that the USA and Chinese distributions are located at sufficiently close values to overlap. Conversely, the probability that μ_{Ψ^α} for France and Italy are at the same position is lower than 5%. If we look at Poland and

[45] It is important to recall that Figure 5 reports the probability that the mean of the original Ψ_k^α distributions is within a certain range, not that the two distributions are bounded by the same limits.

	AUS	AUT	BRA	CAN	CHN	DEU	ESP	FRA	GBR	ITA	IND	JPN	KOR	MEX	POL	TUR	TWN	USA
AUS		0.03	0.36			0.1		0.28	0.26		0.56			1.81	0.16	1.17		
AUT	0.03		0.55		0.11			0.05	0.02		0.8			1.97	0.41	1.35		0.11
BEL	0.2		0.68	0.14	0.29		0.11			0.12	0.99	0.13		2.09	0.6	1.49		0.28
BGR			0.37			0.03		0.2	0.18		0.55			1.8	0.18	1.16		
BRA	0.36	0.55		0.38	0.31	0.6	0.36	0.73	0.72	0.32		0.31	0.51	0.99	0.06	0.35	0.5	0.26
CAN			0.38			0.05		0.22	0.2		0.57			1.82	0.19	1.19		
CHE	0.41	0.15	0.83	0.34	0.51	0.08	0.29			0.29	1.24	0.31	0.14	2.23	0.83	1.64		0.47
CHN		0.11	0.31			0.18		0.36	0.34		0.5		0.06	1.78	0.1	1.13	0.04	
CYP			0.46					0.04	0.02		0.65			1.86	0.28	1.24		0.01
CZE		0.17	0.29			0.24		0.42	0.4		0.51		0.11	1.78	0.09	1.13	0.08	
DEU	0.1		0.6	0.05	0.18		0.02			0.03	0.88	0.05		2.01	0.49	1.41		0.18
DNK			0.39					0.1	0.07		0.54			1.78	0.19	1.16		
ESP			0.36			0.02		0.19	0.17		0.53			1.78	0.16	1.15		
EST		0.14	0.27			0.21		0.39	0.37		0.45		0.09	1.75	0.06	1.1	0.07	
FIN	0.11		0.6	0.05	0.18		0.03			0.05	0.85	0.06		2	0.47	1.39		0.18
FRA	0.28	0.05	0.73	0.22	0.36		0.19			0.2	1.03	0.21	0.05	2.12	0.65	1.52		0.35
GBR	0.26	0.02	0.72	0.2	0.34		0.17			0.17	1.02	0.19	0.02	2.11	0.63	1.51		0.32
GRC	0.33	0.51		0.36	0.28	0.57	0.34	0.69	0.68	0.3		0.29	0.48	0.9	0.04	0.28	0.5	0.24
HRV	0.13		0.62	0.08	0.2		0.06			0.08	0.84	0.08		1.99	0.48	1.38		0.21
HUN		0.1	0.33			0.17		0.36	0.34		0.54		0.05	1.81	0.14	1.16	0.04	
IDN	0.21	0.44		0.24	0.15	0.51	0.21	0.67	0.65	0.17	0.04	0.15	0.39	1.43		0.76	0.32	0.09
IND	0.56	0.8		0.57	0.5	0.88	0.53	1.03	1.02	0.47		0.46	0.73	1.25	0.19	0.56	0.57	0.42
IRL		0.04	0.23			0.1		0.26	0.23		0.35			1.64		1.01	0.03	
ITA			0.32			0.03		0.2	0.17		0.47			1.73	0.11	1.1		
JPN			0.31			0.05		0.21	0.19		0.46			1.72	0.09	1.09		
KOR			0.51		0.06			0.05	0.02		0.73			1.92	0.35	1.3		0.07
LTU	0.2	0.43		0.23	0.15	0.5	0.21	0.66	0.64	0.16	0.04	0.15	0.38	1.42		0.77	0.33	0.09
LUX	0.27	0.04	0.72	0.21	0.34		0.18			0.19	1.02	0.21	0.04	2.11	0.64	1.51		0.33
LVA		0.14	0.3			0.22		0.39	0.37		0.51		0.09	1.78	0.09	1.14	0.06	
MEX	1.81	1.97	0.99	1.82	1.78	2.01	1.78	2.12	2.11	1.73	1.25	1.72	1.92		1.56	0.24	1.8	1.71
MLT	0.07		0.58	0.01	0.15			0.03	0.01		0.86	0.02		2.01	0.46	1.39		0.15
NLD			0.51		0.06			0.07	0.05		0.74			1.93	0.35	1.31		0.06
NOR			0.45					0.1	0.07		0.64			1.86	0.27	1.24		
POL	0.16	0.41	0.06	0.19	0.1	0.49	0.16	0.65	0.63	0.11	0.19	0.09	0.35	1.56		0.89	0.27	0.03
PRT	0.03	0.25		0.07		0.31	0.06	0.45	0.43	0.01	0.05	0.01	0.21	1.39		0.76	0.22	
ROU	0.54	0.75		0.56	0.49	0.81	0.53	0.95	0.93	0.48		0.47	0.7	1.03	0.21	0.36	0.61	0.43
RUS	0.27	0.03	0.74	0.21	0.36		0.18			0.18	1.08	0.2	0.02	2.14	0.68	1.54		0.35
SVK	0.24	0.47		0.27	0.18	0.54	0.25	0.69	0.67	0.2		0.18	0.42	1.38		0.72	0.36	0.12
SVN	0.23		0.69	0.17	0.31		0.14			0.16	0.99	0.16		2.09	0.61	1.48		0.3
SWE		0.05	0.36			0.12		0.3	0.27		0.57			1.82	0.17	1.19		
TUR	1.17	1.35	0.35	1.19	1.13	1.41	1.15	1.52	1.51	1.1	0.56	1.09	1.3	0.24	0.89		1.2	1.07
TWN			0.5		0.04						0.57			1.8	0.27	1.2		0.06
USA		0.11	0.26			0.18		0.35	0.32		0.42		0.07	1.71	0.03	1.07	0.06	

Table 1: 95% credibility bound of the pair-wise difference of posterior distributions of the mean parameters of the total profit-wage ratios for all countries ($\mu^{\alpha} - \mu^{\beta}$). Missing values indicate a possible overlap with a lower than 95% credibility interval for the null. *Source*: authors' calculations.

Germany, it is highly credible that they are completely apart.

To generalize our test about $\Psi_j^A \approx \Psi_k^B$, we report in Table 1 a subset of country's pair-wise credibility tests for 2014 —The full set of country results can be found in Appendix D.4. The table only reports the cases in which the 95% highest density interval excludes zero, that is, the cases that would reject the null hypothesis that the average difference between the credible values of the mean for two distributions is equal to zero, $\mu_{\Psi^A} - \mu_{\Psi^B} = 0$.

Table 1 raises three issues with the IRP-RC hypothesis. First, it indicates that we can credibly reject the hypothesis that two distributions (Ψ_j^A, Ψ_k^B) have the same mean in the majority of cases. Second, there are relevant instances in which the two distributions are extremely close, such as China and the U.S., but there doesn't seem to be a strong correlation between a tight productive or trade interrelation and the proximity of the distributions. For instance, Italy and France, or Germany and China, show no overlap in the probable values of their means. This suggests that the tight proximity between TPWR of some countries is neither a stylized fact nor a predictable result of the likeness, proximity, or integration of different economies. Finally, in many cases, the distance is very large, as was readily observable in Figure 2.

Together, the evidence calls into question the assumption of $\Psi_j^A \approx \Psi_k^B$ so that $\frac{1+\Psi_j^A}{1+\Psi_k^B}$ is an uninformative term centered around one and, therefore, neutral in the long-run determination of IRP.

5 Conclusions and Discussions

This paper has reconstructed and evaluated the theory of international relative prices (IRP) based on the theory of 'real competition' (RC), IRP-RC, developed by Anwar Shaikh and applied to a wide spectrum of economies. The main thesis of the theory is that the long-run behavior of IRP of any pair of tradable commodity bundles is determined exclusively by their relative total unit labor costs (RTULC). This thesis is equivalent to the constraint that the total profits-wages ratios (TPWR) of these two commodity bundles are sufficiently similar at any point in time. To sustain this result, the literature has advanced a set of hypotheses ($\mathcal{H}_1$ and $\mathcal{H}_2$) and arguments ($\mathcal{A}_1 - \mathcal{A}_5$) which, according to its proponents, are rooted in the classical theory competition.

The theoretical and empirical results of our paper cast doubts on the soundness of the theory of IRP-RC. On the one hand, due to accounting reasons, the proposed hypotheses are *inefficient* to produce the desire thesis: Proposition 2 showed that the assertion that the terms of trade of any pair of tradable commodities equal their RTULC requires world uniformity in TPWR, $\Psi_j^A = \Psi_k^B = \Psi$, not *within country* uniformity $\Psi_j^A = \Psi^A$ and $\Psi_k^B = \Psi^B$, as sustained by the theory. For the real exchange rate of any pair of traded commodity bundles to equal their RTULC, Proposition 4 shows that we need a special combination of industrial TPWR of different countries (Ψ_j^A, Ψ_k^B) and composition of the traded commodity baskets (d^A, d^B) that forces $\Psi_d^A = \Psi_d^B$. Not only these conditions were not identified in the literature but they are also at odds with the plead sharp cross-country differences in technology and distribution in their critique of the neoclassical theory of international trade.

On the other hand, the theoretical and empirical arguments used to constrain the TPWR are *weak* due to their speculative nature. We showed that none of Argument $\mathcal{A}_1$ to $\mathcal{A}_5$ and the forces of competition constrain the TPWR in such a way to produce the advanced nor the needed hypotheses to derive the desired thesis.

We showed that the knowledge of the literature on the statistical properties of TPWR of actual economies is almost empty. Given this vacuum of knowledge, we conducted the first large-scale empirical study of the TPWR, capital intensities, and wage-shares based on the WIOD and PWT databases. It was found that the functional form of the empirical densities of these variates across industries-within countries $\Psi_1^\alpha, \ldots, \Psi_n^\alpha$ and $\frac{K_1^\alpha}{v_1^\alpha}, \ldots, \frac{K_n^\alpha}{v_n^\alpha}$ and across countries-within industries $\Psi_j^{\text{AUS}}, \ldots, \Psi_j^{\text{USA}}$ and $\frac{K_j^{\text{AUS}}}{v_j^{\text{AUS}}}, \ldots, \frac{K_j^{\text{USA}}}{v_j^{\text{USA}}}$ display high homogeneity characterized by a strong clustering of the data around a single mode with limited variability characterized by a slight skewness towards lower values and a fat right-hand tail. We used Bayesian methods to study the posterior probability distribution of the means of these TPWR and found that, in spite of the former stylized facts, except for some pairs of countries, e.g., China and the U.S., pairs of (Ψ_j^A, Ψ_k^B) could differ considerably, casting doubts on the long-run neutrality of the TPWR in the determination of IRP in general.

How our theoretical and empirical results relate to (i) the empirical evidence on the long-run association between IRP and RTULC from the literature and (ii) the theory of IRP based on the principle of absolute advantage under long-period positions?

Regarding (i), the only support to the theory of IRP-RC that we have detected is the reported empirical evidence on the long-run association between IRP and their RTULC. However, these regularities cannot be explained by the theory and, therefore, constitute now a conundrum within the literature. Now, our paper has not evaluated the empirical association between IRP and RTULC reported by the literature, but the lack of sound economic arguments and limited empirical evidence to sustain $\Psi_j^A \approx \Psi_k^B$ suggest that this literature needs revision. In addition, the existence of a long-run association between IRP and RTULC does not mean that techno-distributive information embedded in Ψ_j^α is neutral. This literature adopts the hypothesis of inter-country homogeneity in TPWR, i.e., $\Psi_j^A = \Psi^A$ and $\Psi_k^B = \Psi^B$. Hence, Proposition 3 and 5 points that one immediate statistical exercise is to include $\frac{1+\psi^A}{1+\psi^B}$ in their regressions.[46]

As for (ii), we want to discuss two implications. Firstly, with no reasons to dismiss the techno-distributive elements contained in the relative TPWR we need to consider *full* production prices in the study of the long-run behavior of IRP, not only the RTULC. For this, the study of the causes and effects of IRP's levels and dynamics can be conducted using open-economy Sraffian models and/or Pasinetti's (1981) structural dynamics models. Under these theoretical schemes, it would be relevant to conduct an evaluation of the role played by the commodity bundles (d^A, d^B) in the determination of IRP, something that has been ignored by the literature on the theory of IRP-RC.

Second, the persistent and generalized statistical characteristics of the Ψ_j and $\frac{K_j}{v_j}$ add to the regularities found in other features of the productive and allocative structure in empirical linear production models.[47] While the empirical regularities on Ψ_j and $\frac{K_j}{v_j}$ cast doubt on the hypothesis required to sustain the main thesis of the theory of IRP-RC, it also rejects the uninformative structure of the techniques of production and social compositions of output implicit in typical linear production models. These models are general enough to proof the existence of economically meaningful prices and quantities, but cannot explain their empirical regularities. The statistical structure of the Ψ_j and $\frac{K_j}{v_j}$ indicates the existence of relevant technological and distributive constraints. We need to find a middle ground as has been found in the literature on the explanation on the empirical regularities in price- and wage-profits curves.[48]

[46] Assuming with this that the relative $\sum_{i=1}^n \sigma_i^\alpha m_{ij}^\alpha$ in (15) and (18) are uninformative components. Fevereiro (2019) studies the relationship between the wage-share and the undervaluation of the RER.

[47] Mariolis and Tsoulfidis (2011) and Shaikh et al. (2023) report robust statistical regularities on the eigenvalues of the input-coefficient matrices and Torres-González and Yang (2019) for their column and row sums as well as in their left and right Perron-Frobenius (P-F) eigenvectors. Torres-González (2022) and Ferrer-Hernández and Torres-González (2022) report a statistical tendency towards the proportionality between the labor vector and the left P-F eigenvector. Torres-González (2022) finds results consistent to those in Section 4 for the U.S. economy in 1977-2012 (between 402-530 industries).

[48] See Mariolis and Tsoulfidis (2011), Schefold (2013, 2023), and Ferrer-Hernández and Torres-González 2022.

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A The construction of the total profits-wages ratios Ψ_j and their alternative representations

A.1 Obtaining Equation (3)

Let x_j be the value of output of industry j. From the perspective of cost accounting, this market value can be decomposed into

$$x_j = \text{wages}_j + \text{profits}_j + \text{value of means of production}_j. \tag{A.1}$$

Value added is represented by $\text{wages}_j + \text{profits}_j$ whereas intermediate consumption by the value of means of production_j. Under the assumptions of the economy given in 2.1, the value of output of any industry can be expressed as $x_j = p_j q_j$, where p_j is the market price of commodity j and q_j the quantity of this commodity produced. Hence,

$$p_j = \omega_j + \pi_j + \lambda_j \quad \text{for} \quad j \in N, \tag{A.2}$$

where $\omega_j \equiv \frac{\text{wages}_j}{q_j} > 0$,[A.1] $\pi_j \equiv \frac{\text{profits}_j}{q_j} \geqslant 0$, and $\lambda_j \equiv \frac{\text{value of means of prod.}_j}{q_j} \geqslant 0$ represent the labor costs, profits, and the means of production costs in industry j, all of them per unit of output j. These inequalities imply that value added in each industry is positive.

Now, the unit value of the means of production λ_j can be further decomposed into the wages $\omega_j^{(1)}$, profits $\pi_j^{(1)}$, and value of the means of production $\lambda_j^{(1)}$ incurred by *all the suppliers to industry j needed to produce one unit of commodity j*,

$$\lambda_j = \omega_j^{(1)} + \pi_j^{(1)} + \lambda_j^{(1)} \quad j \in N. \tag{A.3}$$

But this process can be done now for the suppliers of the suppliers of commodity j, and so on and so forth, so for $j \in N$ we have

$$\lambda_j^{(1)} = \omega_j^{(2)} + \pi_j^{(2)} + \lambda_j^{(2)} \tag{A.4}$$

$$\lambda_j^{(2)} = \omega_j^{(3)} + \pi_j^{(3)} + \lambda_j^{(3)} \tag{A.5}$$

$$\vdots \tag{A.6}$$

$$\lambda_j = \omega_j^{(1)} + \pi_j^{(1)} + \omega_j^{(2)} + \pi_j^{(2)} + \omega_j^{(3)} + \pi_j^{(3)} + \cdots + \omega_j^{(\tau)} + \pi_j^{(\tau)} + \lambda_j^{(\tau)} \tag{A.7}$$

$$= \sum_{\nu=1}^{\tau-1} (\omega_j^{(\nu)} + \pi_j^{(\nu)}) + (\omega_j^{(\tau)} + \pi_j^{(\tau)} + \lambda_j^{(\tau)}) \tag{A.8}$$

[A.1] Labor is indispensable.

Let $\{\omega_j^{(\nu)}\}$, $\{\pi_j^{(\nu)}\}$, and $\{\lambda_j^{(\nu)}\}$ be a sequence for $\nu = 0, 1, 2, \ldots$, each ν representing a specific segment of the value chain of commodity j. Set $\omega_j^{(0)} \equiv \omega_j$, $\pi_j^{(0)} \equiv \pi_j$, and $\lambda_j^{(0)} \equiv \lambda_j$. Because value added is positive for each ν, i.e., $\omega_j^{(\nu)} + \pi_j^{(\nu)} = \lambda_j^{(\nu)} - \lambda_j^{(\nu+1)} > 0$, this means that the sequence $\{\lambda_j^{(\nu)}\}$ is strictly decreasing, $\lambda_j^{(\nu)} > \lambda_j^{(\nu+1)}$. In addition, the sequence $\{\lambda_j^{(\nu)}\}$ is bounded from bellow, $\lambda_j^{(\nu)} \geqslant 0$. This means that the sequence $\{\lambda_j^{(\nu)}\} = \{\omega_j^{(\nu+1)} + \pi_j^{(\nu+1)} + \lambda_j^{(\nu+1)}\}$ converges to zero, i.e, $\lim_{\nu\to\infty} \lambda_j^{(\nu)} = \omega_j^{(\nu+1)} + \pi_j^{(\nu+1)} + \lambda_j^{(\nu+1)} = 0$, and therefore $\sum_{k=1}^{\infty}(\omega_j^{(\nu)} + \pi_j^{(\nu)}) < \infty$.[A.2] Hence, for $j \in N$ we have

$$\lambda_j = \sum_{\nu=1}^{\infty} \omega_j^{(\nu)} + \sum_{\nu=1}^{\infty} \pi_j^{(\nu)} = \omega_j^I + \pi_j^I, \tag{A.9}$$

where $\omega_j^I \equiv \sum_{\nu=1}^{\infty} \omega_j^{(\nu)}$ and $\pi_j^I \equiv \sum_{\nu=1}^{\infty} \pi_j^{(\nu)}$. Substituting (A.9) into (2) we can get (3):

$$p_j = (\omega_j + \omega_j^I) + (\pi_j + \pi_j^I) = \Omega_j + \Pi_j \tag{A.10}$$

where $\Omega_j \equiv \omega_j + \omega_j^I = \sum_{\nu=0}^{\infty} \omega_j^{(\nu)}$ is the *total* or vertically integrated unit labor costs, $\Pi_j \equiv \pi_j + \pi_j^I = \sum_{\nu=0}^{\infty} \pi_j^{(\nu)}$ are the total unit profits.

A.2 Shaikh's representation of Ψ_j

Shaikh (1984, p. 68) provides the decomposition of total profits-wages ratios Ψ_j as an infinite convex combination of *average* direct profit-wages ratios in each stage of the value chain of a commodity $\psi_j^{(\nu)}$ with weights given by the shares of the direct labor in stage ν into total labor $\frac{l_j^{(\nu)}}{v_j}$, i.e.,

$$\Psi_j = \psi_j \frac{l_j}{v_j} + \psi_j^{(1)} \frac{l_j^{(1)}}{v_j} + \psi_j^{(2)} \frac{l_j^{(2)}}{v_j} + \cdots, \tag{A.11}$$

where $\psi_j^{(\nu)} \equiv \frac{\pi_j^{(\nu)}}{\omega_j^{(\nu)}}$, $l_j^{(\nu)} \equiv \frac{L_j^{(\nu)}}{q_j}$, $L_j^{(\nu)}$ is the quantity of labor needed in the production of means of production in stage ν in order to produce q_j, $v_j \equiv \frac{V_j}{q_j}$, and $V_j = \sum_{\nu=0}^{\infty} L_j^{(\nu)}$ is the total (or direct plus indirect) quantity of labor needed in the production q_j.

To obtain (A.11) we first define the total profits-wages ratios:

$$\Psi_j \equiv \frac{\Pi_j}{\Omega_j}. \tag{A.12}$$

Second, assume long-period positions for labor.[A.3] Hence, we can express wages per unit of output of industry j in any stage of the value chain as $\omega_j^{(\nu)} = \frac{wL_j^{(\nu)}}{q_j}$ for

[A.2] See Nikaido (1968, Theorem 15.2, p. 112).
[A.3] See Section 2.5.

$\nu = 0, 1, 2, \ldots$. Previously we defined $l_j^{(\nu)} \equiv \frac{L_j^{(\nu)}}{q_j}$ and $v_j \equiv \frac{V_j}{q_j}$. Hence,

$$\omega_j^{(\nu)} = w l_j^{(\nu)} \tag{A.13}$$

$$\Omega_j = \sum_{\nu=0}^{\infty} \omega_j^{(\nu)} = w \sum_{\nu=0}^{\infty} l_j^{(\nu)} = w v_j \tag{A.14}$$

Finally, given (A.12), (A.14), $\Pi_j = \sum_{\nu=0}^{\infty} \pi_j^{(\nu)}$, and $\psi_j^{(\nu)} \equiv \frac{\pi_j^{(\nu)}}{\omega_j^{(\nu)}} = \frac{\pi_j^{(\nu)}}{w l_j^{(\nu)}}$ we have

$$\Psi_j = \frac{\pi_j + \pi_j^{(1)} + \pi_j^{(2)} + \cdots}{w v_j} \tag{A.15}$$

$$= \frac{\pi_j}{w v_j} + \frac{\pi_j^{(1)}}{w v_j} + \frac{\pi_j^{(2)}}{w v_j} + \cdots \tag{A.16}$$

$$= \frac{\pi_j}{w v_j} \frac{w l_j}{w l_j} + \frac{\pi_j^{(1)}}{w v_j} \frac{w l_j^{(1)}}{w l_j^{(1)}} + \frac{\pi_j^{(2)}}{w v_j} \frac{w l_j^{(2)}}{w l_j^{(2)}} + \cdots \tag{A.17}$$

$$= \frac{\pi_j}{w l_j} \frac{l_j}{v_j} + \frac{\pi_j^{(1)}}{w l_j^{(1)}} \frac{l_j^{(1)}}{v_j} + \frac{\pi_j^{(2)}}{w l_j^{(2)}} \frac{l_j^{(2)}}{v_j} + \cdots \tag{A.18}$$

$$= \psi_j \frac{l_j}{v_j} + \psi_j^{(1)} \frac{l_j^{(1)}}{v_j} + \psi_j^{(2)} \frac{w L_j^{(2)}}{w L_j^{(2)}} + \cdots \tag{A.19}$$

$\psi^{(\nu)} \equiv \frac{\pi_j^{(\nu)}}{w l_j^{(\nu)}}$ is the ratio between the profits and wages of the industries participating in the ν stage of the value chain, and $\frac{l_j^{(\nu)}}{v_j}$ is the share of the indirect labor contained in stage ν of the value chain over total labor contained in the production of commodity j. Notice that each $\psi^{(\nu)}$ is a weighted average of the direct profits-wages ratios only of the industries participating in the stage ν of the value chain. Because the value chain is potentially composed of an infinite number of stages, this representation of Ψ_j (1) requires an infinite number of weighted averages of the direct profits-wages and (2) aggregates direct profits-wages ratios of different industries, so it is not possible to identify the specific contribution of each industry.

A.3 Obtaining Equation (5) from a linear production model - The Total Profits-Wages Ratios Ψ_j as a Convex Combination of the Direct Profits-Wages Ratios ψ_j

Our proposed decomposition of total profit-wages ratios $\Psi_j = \frac{\Pi_j}{\Omega_j}$ in (4) is

$$\Psi_j = \frac{\Pi_j}{\Omega_j} = \psi_1 m_{1j} + \psi_2 m_{2j} + \cdots + \psi_n m_{nj} \quad \text{for} \quad j \in N. \tag{A.20}$$

Let us now derive the weights $m_{1j}, m_{2j}, \ldots, m_{nj}$ and obtain (A.20). We now do this using a linear production model. In particular, we use Pasinetti (1973, Section 2) production model where fixed capital is treated as a durable means of production with uniform exponential rates of depreciation for each commodity-input within each industry.

Let us start with (A.1) and $x_j = p_j q_j$. The value of the means of production of commodities $j \in N$ can be decomposed as price-times-quantity magnitudes:

$$\text{value of means of production}_j = \sum\nolimits_{i=1}^{n} p_i Q_{ij} + p_i D_{ij} = \sum\nolimits_{i=1}^{n} p_i (Q_{ij} + D_{ij}), \tag{A.21}$$

where p_j is the market price of the j-th commodity, q_j is the quantity produced of commodity j, Q_{ij} is the quantity of commodity i used as input in the production of commodity j, and D_{ij} is the quantity of durable commodity i depreciated in industry j. Notice that (i) industries do not use imported means of production, (ii) there are no assumptions on the input-output relations, and (iii) there are no constraints on the labor inputs and the composition of the capital advanced as well as in their rates of return.

Now, consider the following options: (a) the quantities q_j, Q_{ij}, and D_{ij}, for $i, j \in N$, are given and *define* the ratios $a_{ij} \equiv \frac{Q_{ij}}{q_j}$ and $d_{ij} \equiv \frac{D_{ij}}{q_j}$ for $i, j \in N$, respectively; or (2) there is proportionality between commodity means of production and depreciation and outputs, $Q_{ij} = a_{ij} q_j$ and $D_{ij} = d_{ij} q_j$ for $i, j \in N$, respectively. Assuming either of the two options we can express (2) as

$$\begin{aligned} p_j &= \sum\nolimits_{i=1}^{n} p_i \frac{Q_{ij} + D_{ij}}{q_j} + \frac{\text{wages}_j}{q_j} + \frac{\text{profits}_j}{q_j} \\ &= \sum\nolimits_{i=1}^{n} p_i a_{ij} + \sum\nolimits_{i=1}^{n} p_i d_{ij} + \omega_j + \pi_j \quad \text{for} \quad j \in N. \end{aligned} \tag{A.22}$$

Expressing (A.22) in matrix notation we have:

$$\begin{aligned} \mathbf{p}' &= \mathbf{p}'(\mathbf{A} + \mathbf{D}) + \boldsymbol{\omega}' + \boldsymbol{\pi}' = \mathbf{p}'\mathbf{A}^{\ominus} + \boldsymbol{\omega}' + \boldsymbol{\pi}' \\ \mathbf{p}' \left(\mathbf{I} - \mathbf{A}^{\ominus}\right) &= \boldsymbol{\omega}' + \boldsymbol{\pi}', \end{aligned} \tag{A.23}$$

where $\underset{(1\times n)}{\mathbf{p}}{}' \equiv [p_j]$ is the price vector, $\underset{(n\times n)}{\mathbf{A}} \equiv [a_{ij}] \equiv [\frac{Q_{ij}}{q_j}] \geqq \mathbf{0}$ is the physical input-coefficients matrix, $\underset{(n\times n)}{\mathbf{D}} \equiv [d_{ij}] \equiv [\frac{D_{ij}}{q_j}] \geqq \mathbf{0}$ is the physical depreciation-coefficients matrix, $\mathbf{A}^{\ominus} \equiv \mathbf{A} + \mathbf{D}$, $\underset{(1\times n)}{\boldsymbol{\omega}'} \equiv [\omega_j]$ is the vector with the direct wages per unit of output, and $\underset{(1\times n)}{\boldsymbol{\pi}'} \equiv [\pi_j]$ is the vector with the direct profits per unit of output.

Assume that $\mathbf{A}^{\ominus}$ is productive (viable) so $(\mathbf{I}-\mathbf{A}^{\ominus})$ is non-singular and $(\mathbf{I}-\mathbf{A}^{\ominus})^{-1} = \mathbf{I}+\sum_{k=1}^{\infty}(\mathbf{A}^{\ominus})^k \geqslant \mathbf{0}$ is semipositive with a positive diagonal.[A.4] Matrix $\boldsymbol{\Lambda} \equiv (\mathbf{I}-\mathbf{A}^{\ominus})^{-1} \equiv [\Lambda_{ij}]$ is called Leontief's inverse. Then, solving (A.23) for $\mathbf{p}$ we get

$$\mathbf{p}' = \boldsymbol{\omega}'\boldsymbol{\Lambda} + \boldsymbol{\pi}'\boldsymbol{\Lambda} = \boldsymbol{\Omega}' + \boldsymbol{\Pi}', \tag{A.24}$$

where $\boldsymbol{\omega}'\boldsymbol{\Lambda} \equiv \boldsymbol{\Omega}' \equiv [\Omega_j]$ is the vector with the total wages per unit of output, and $\boldsymbol{\pi}'\boldsymbol{\Lambda} \equiv \boldsymbol{\Pi}' \equiv [\Pi_j]$ is the vector with the total profits per unit of output.

Let any row vector with 'hat' be a squared diagonal matrix with the elements of that vector in the main diagonal. Hence $\hat{\boldsymbol{\Omega}} = \mathrm{diag}\{\Omega_1, \ldots, \Omega_n\}$ and $\hat{\boldsymbol{\Omega}}^{-1} = \mathrm{diag}\{\frac{1}{\Omega_1}, \ldots, \frac{1}{\Omega_n}\}$. Notice that $\boldsymbol{\omega}' = \mathbf{1}'\hat{\boldsymbol{\omega}}$, where $\mathbf{1}' \equiv [1]$ is the summation vector —a vector with ones. With these definitions, and remembering that $\boldsymbol{\Omega}' = \boldsymbol{\omega}'\boldsymbol{\Lambda}$, we can obtain the matrix with the weights m_{ij} for $i, j \in N$ which will be used in the construction of the total profits-wages ratios:

$$\mathbf{1}' = \boldsymbol{\Omega}'\hat{\boldsymbol{\Omega}}^{-1} = \boldsymbol{\omega}'\boldsymbol{\Lambda}\hat{\boldsymbol{\Omega}}^{-1} = \mathbf{1}'\hat{\boldsymbol{\omega}}\boldsymbol{\Lambda}\hat{\boldsymbol{\Omega}}^{-1} = \mathbf{1}'\mathbf{M}, \tag{A.25}$$

where $\hat{\boldsymbol{\omega}}\boldsymbol{\Lambda}\hat{\boldsymbol{\Omega}}^{-1} \equiv \mathbf{M} \equiv [m_{ij}] \geqslant \mathbf{0}$ is the matrix with weights in (5) the column sums of which are equal to one. Notice that $m_{ij} = \frac{\omega_i \Lambda_{ij}}{\Omega_j}$. Because

$$\mathbf{1}'\mathbf{M} = \mathbf{1}', \tag{A.26}$$

then $\mathbf{M}$ is a column stochastic matrix with a dominant eigenvalue $\rho(\mathbf{M}) = 1$ associated with the positive eigenvector $\mathbf{1}'$. Hence, the columns of matrix $\mathbf{M}$ inform us on the location of the indirect or direct unit labor cost in every industry which conforms the total unit labor costs in industry j. Finally, under long-period position the wage rate in the numerator and denominator of each $m_{ij} = \frac{\omega_i \Lambda_{ij}}{\Omega_j}$ is canceled and $\mathbf{M} = \hat{\mathbf{l}}\boldsymbol{\Lambda}\hat{\mathbf{v}}^{-1}$, where $m_{ij} = \frac{l_i \Lambda_{ij}}{v_j}$ and $v_j = \sum_{i=1}^{n} l_i \Lambda_{ij}$.

Now, we can express (A.24) as in (3):

[A.4] If $\mathbf{A}^{\ominus}$ is in addition irreducible, then $(\mathbf{I} - \mathbf{A}^{\ominus})^{-1} > \mathbf{0}$ (Takayama, 1974, p. 392).

$$\mathbf{p}' = \mathbf{\Omega}' + \mathbf{\Pi}' = \mathbf{\Omega}'(\mathbf{I} + \hat{\mathbf{\Pi}}\hat{\mathbf{\Omega}}^{-1}) = \mathbf{\Omega}'(\mathbf{I} + \hat{\mathbf{\Psi}}), \tag{A.27}$$

where $\hat{\mathbf{\Pi}}\hat{\mathbf{\Omega}}^{-1} \equiv \hat{\mathbf{\Psi}} \equiv \mathrm{diag}\{\frac{\Pi_1}{\Omega_1}, \ldots, \frac{\Pi_n}{\Omega_n}\}$ is the diagonal matrix with the total profits-wages ratios. Define the vector with the *direct* profits-wages ratios as $\boldsymbol{\pi}'\hat{\boldsymbol{\omega}}^{-1} \equiv \boldsymbol{\psi}' \equiv [\psi_j] \equiv [\frac{\pi_j}{\omega_j}]$. This means that $\boldsymbol{\pi}' = \boldsymbol{\psi}'\hat{\boldsymbol{\omega}}$. Remembering that $\mathbf{\Pi}' \equiv \boldsymbol{\pi}'\mathbf{\Lambda}$, then

$$\mathbf{\Psi}' \equiv \mathbf{1}'\hat{\mathbf{\Psi}} = \mathbf{1}'\hat{\mathbf{\Pi}}\hat{\mathbf{\Omega}}^{-1} = \mathbf{\Pi}'\hat{\mathbf{\Omega}}^{-1} = \boldsymbol{\pi}'\mathbf{\Lambda}\hat{\mathbf{\Omega}}^{-1} = \boldsymbol{\psi}'\hat{\boldsymbol{\omega}}\mathbf{\Lambda}\hat{\mathbf{\Omega}}^{-1} = \boldsymbol{\psi}'\mathbf{M}, \tag{A.28}$$

$$\Psi_j = \boldsymbol{\psi}'\mathbf{M}_{(j)} = \sum_{i=1}^{n} \psi_i m_{ij} \quad \text{for} \quad j \in N, \tag{A.29}$$

so Ψ_j is a convex linear combination, or a weighted average, with the $\{m_{1j}, \ldots, m_{nj}\}$ as weights, consistent with (4).

Notice that although the same n ratios ψ_i enter in each Ψ_j, the weights m_{ij} in which the ψ_i enter in each Ψ_j might be different in each industry. Therefore, $\Psi_j = \Psi_k$ for $j, k \in N$ requires n weighting systems such that the n weighted averages of the same direct profits-wages ratios $\sum_{i=1}^{n} \psi_i m_{ij}$ are all equal. On the other hand, if all the *direct* profits-wages ratios are the same $\psi_j = \psi$, then $\Psi_j = \psi \sum_{i=1}^{n} m_{ij} = \psi$ for any weighting system. Hence, small variability in the direct profits-wages ratios ψ_j coupled with a set of weights that maintains or reduces this variability can contribute to maintain small variability in the total profits-wages ratios Ψ_j.

We identify the following advantages of our decomposition (A.20) vis-a-vis the one proposed by Shaikh (1984) (A.11). Firstly, every Ψ_j depends on the same n *direct* ratios ψ_j, so the contributions of each ψ_j can be identified. Second, given that $m_{ij} = \frac{l_i \Lambda_{ij}}{v_j}$, the weights $(m_{1j}, m_{2j}, \ldots, m_{nj})$ for each Ψ_j represent the share of the total labor conducted in industry i, $l_i \Lambda_{ij}$, in the total labor of the society in order to produce one unit of net output of commodity j, $v_j = \sum_{i=1}^{n} l_i \Lambda_{ij}$. Third, there is a finite number of ψ_j and $(m_{1j}, m_{2j}, \ldots, m_{nj})$ for each Ψ_j.

A.4 Some statistical properties of Ψ_j

We now show that Ψ_j depends on the statistical characteristics of *all* the direct ratios ψ_j, the weights m_{ij} for $i \in N$, and their relationship. Given that for any two random variables $\mathrm{Cov}(X, Y) = E(XY) - E(X)E(Y)$ and that $\bar{m}_j \equiv \frac{1}{n} \sum_{i=1}^{n} m_{ij} = \frac{1}{n}$ for $j \in N$, then we have:

$$\Psi_j \equiv \sum_{i=1}^{n} \psi_i m_{ij} = n\mathrm{Cov}(\psi_i, m_{ij}) + \bar{\psi} \quad \text{for} \quad j \in N, \tag{A.30}$$

where $\bar{\psi} \equiv \frac{1}{n}\sum_{i=1}^{n}\psi_i$ is a simple average of all industries and does not necessarily represent the economy-wide profits-wages ratio ψ in (7). Given that $\bar{\psi}$ in (A.30) is the same for each Ψ_j, the variability of the Ψ_j depends then on the covariance between the direct ψ_i and the weights m_{ij}. But the same n ratios ψ_i participate in each Ψ_j, so the more similar the n columns $(m_{1j}, m_{2j}, \ldots, m_{nj})$ of matrix $\mathbf{M}$ are, the more similar the n ratios Ψ_j will be.

Equation (9) shows that the $(1 + \Psi_j)$ can be made dependent on an *economy-wide average* $(1+\psi)$ and, just like in (A.30), a 'covariance-plus-simple average' decomposition of an industry-level weighted average:

$$(1 + \Psi_j) = (1 + \psi)\sum_{i=1}^{n}\sigma_i m_{ij} = (1 + \psi)\left[n\mathrm{Cov}(\sigma_i, m_{ij}) + \bar{\sigma}\right] \quad \text{for} \quad j \in N, \tag{A.31}$$

where $\bar{\sigma} \equiv \frac{1}{n}\sum_{i=1}^{n}\sigma_i = \frac{1+\bar{\psi}}{1+\psi}$. The distance of $\bar{\sigma}$ from 1 measures the difference in industries weights from uniform weights $\frac{1}{n}$.

B Construction of the database

B.1 The World Input-Output Database

The World Input-Output Database (WIOD), in its 2016 release, constructed by Timmer et al. (2015), provides estimates of annual time-series of input–output tables (IOTs) covering 43 countries — 28 EU countries and 15 other major countries in the world — for the period from 2000 to 2014. The country sample represents up to 86% of the world economy in 2016 and includes rich- and middle-income economies of diverse industrial structures and development profiles — least developed countries are not included in the sample. The list of the 43 economies, and their acronym used throughout the document (in parenthesis), is given in table B.1. We decided to exclude Malta from the sample.

The base for the construction of the IOTs are the supply and use tables (SUTs), which are obtained from official national sources and are adapted to a 56 industries common disaggregation detail based on the 2008 System of National Accounts (SNA2008) framework. The input years and the number of releases for which SUTs are available are uneven and dispersed with the base methodology drawing from the SNA2008,

Australia (AUS) [49]	Austria (AUT) [54]	Belgium (BEL) [54]	Bulgaria (BGR) [54]
Brazil (BRA) [47]	Canada (CAN) [51]	Switzerland (CHE) [49]	China, People's Republic of (CHN) [47]
Cyprus (CYP) [54]	Czech Republic (CZE) [54]	Germany (DEU) [54]	Denmark (DNK) [54]
Spain (ESP) [54]	Estonia (EST) [54]	Finland (FIN) [54]	France (FRA) [54]
United Kingdom of Great Britain and Northern Ireland (GBR) [54]	Greece (GRC) [54]	Croatia (HRV) [54]	Hungary (HUN) [54]
Indonesia (IDN) [47]	India (IND) [45]	Ireland (IRL) [54]	Italy (ITA) [54]
Japan (JPN) [50]	Republic of Korea (KOR) [53]	Lithuania (LTU) [54]	Luxembourg (LUX) [52]
Latvia (LVA) [54]	Mexico (MEX) [52]	Malta (MLT) [52]	Netherlands (NLD) [54]
Norway (NOR) [54]	Poland (POL) [54]	Portugal (PRT) [54]	Romania (ROU) [54]
Russian Federation (RUS) [33]	Slovakia (SVK) [54]	Slovenia (SVN) [54]	Sweden (SWE) [53]
Turkey (TUR) [46]	Taiwan (TWN) [54]	United States (USA) [54]	

Table B.1: List of the 43 countries included in the WIOD database, 2016 release, and number of industries for which there is information available in each country-year.

SNA1993, and International System of Industrial Classification, Revision 3, frameworks. From this information, world industry-by-industry IOTs are constructed from which we obtain the IOTs for each country-year.

The WIOD includes fictitious industries which are statistical artifacts to balance the tables. We decide to omit industries T ("Activities of households as employers; undifferentiated goods- and services- producing activities of households for own use") and U ("Activities of extraterritorial organizations and bodies"), whose entries are mostly zeros. Table B.2 gives the list of the final 54 industries. However, not every country in the WIOD has information for the 54 industries. *Our calculations do not consider the zeros from these industries.* The number of industries with nonzero information for each country is given in squared brackets in Table B.1. The number of countries with information for each industry is given in squared brackets in Table B.2. The country with less information is Russia (with 31 industries), but most of the economies fluctuate between 52 and 54 industries.

The WIOD provides in addition socio-economic accounts (SEAs) containing industry-level data, under the same industry classification system as the WIOTs, on the uses of primary inputs (capital and labour), intermediate inputs, gross output, and the components of value added at current and constant prices. Table B.3 provides the full

ISIC4 code	Sector description
A01	Crop and animal production, hunting and related service activities
A02	Forestry and logging
A03	Fishing and aquaculture
B	Mining and quarrying
C10-C12	Manufacture of food products, beverages and tobacco products
C13-C15	Manufacture of textiles, wearing apparel and leather products
C16	Manufacture of wood and of products of wood and cork, except furniture; straw and plaiting materials
C17	Manufacture of paper and paper products
C18	Printing and reproduction of recorded media
C19	Manufacture of coke and refined petroleum products
C20	Manufacture of chemicals and chemical products
C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
C22	Manufacture of rubber and plastic products
C23	Manufacture of other non-metallic mineral products
C24	Manufacture of basic metals
C25	Manufacture of fabricated metal products, except machinery and equipment
C26	Manufacture of computer, electronic and optical products
C27	Manufacture of electrical equipment
C28	Manufacture of machinery and equipment n.e.c.
C29	Manufacture of motor vehicles, trailers and semi-trailers
C30	Manufacture of other transport equipment
C31-C32	Manufacture of furniture; other manufacturing
C33	Repair and installation of machinery and equipment
D35	Electricity, gas, steam and air conditioning supply
E36	Water collection, treatment and supply
E37-E39	Sewerage; waste collection, treatment and disposal activities; materials recovery; other waste services
F	Construction
G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
G46	Wholesale trade, except of motor vehicles and motorcycles
G47	Retail trade, except of motor vehicles and motorcycles
H49	Land transport and transport via pipelines
H50	Water transport
H51	Air transport
H52	Warehousing and support activities for transportation
H53	Postal and courier activities
I	Accommodation and food service activities
J58	Publishing activities
J59-J60	Motion picture, video and television production, sound and music; programming and broadcasting
J61	Telecommunications
J62-J63	Computer programming, consultancy and related activities; information service activities
K64	Financial service activities, except insurance and pension funding
K65	Insurance, reinsurance and pension funding, except compulsory social security
K66	Activities auxiliary to financial services and insurance activities
L68	Real estate activities
M69-M70	Legal and accounting activities; activities of head offices; management consultancy activities
M71	Architectural and engineering activities; technical testing and analysis
M72	Scientific research and development
M73	Advertising and market research
M74-M75	Other professional, scientific and technical activities; veterinary activities
N	Administrative and support service activities
O84	Public administration and defence; compulsory social security
P85	Education
Q	Human health and social work activities
R-S	Other service activities

Table B.2: List with the 54 industries from the WIOD, 2016 release, considered in the sample. *Notes:* Omitted industries are ‘Activities of households as employers; undifferentiated goods- and services- producing activities of households for own use’ (T) and ‘Activities of extraterritorial organisations and bodies’ (U).

Symbol	Variable	Units
GO	Gross output by industry at current basic prices	in millions of national currency
II	Intermediate inputs at current purchasers' prices	in millions of national currency
VA	Gross value added at current basic prices	in millions of national currency
EMP	Number of persons engaged	thousands
EMPE	Number of employees	thousands
H_EMPE	Total hours worked by employees	millions
COMP	Compensation of employees	in millions of national currency
LAB	Labour compensation	in millions of national currency
CAP	Capital compensation	in millions of national currency
$\mathcal{K}$	Nominal capital stock	in millions of national currency
GO_PI	Price levels gross output	2010=100
II_PI	Price levels of intermediate inputs	2010=100
VA_PI	Price levels of gross value added	2010=100
GO_QI	Gross output, volume indices	2010=100
II_QI	Intermediate inputs, volume indices	2010=100
VA_QI	Gross value added, volume indices	2010=100

Table B.3: List of the variables in the Socio-Economic Accounts in the WIOD database, 2016 release, and their description.

description of the available information. A comprehensive overview of the sources and methodological choices for the original release can be found in Dietzenbacher et al. (2013). The labor compensation includes wages and salaries paid to employees and the self-employed in each industry, whose earnings are imputed using the average wages prevalent in the sector (Dietzenbacher et al., 2013).

The information for the national and international industry-by-industry and supply and use tables corresponds to current market international dollar prices. The value data of the SAEs are denoted in millions of national currency. Values were converted into dollars using the exchange rates provided in the WIOD as an independent file.

B.2 The the methodology to compute the total profits-wages ratios and total capital intensities

Whereas the *direct* measures of unit labor cost, unit profits, capital advanced, and unit labor can be calculated directly from the data from the WIOD, for the computation of their total or vertically integrated we employed the same approach as in (A.24) to obtain $\boldsymbol{\omega}'\boldsymbol{\Lambda} \equiv \boldsymbol{\Omega}' \equiv [\Omega_j]$ and $\boldsymbol{\pi}'\boldsymbol{\Lambda} \equiv \boldsymbol{\Pi}' \equiv [\Pi_j]$.

Leontief input-coefficients matrix and inverse. For each country-year we sum industry j's purchases of domestic z_{ij}^D and imported z_{ij}^M intermediate inputs from industry i at market prices, $z_{ij} = z_{ij}^D + z_{ij}^M$, to construct matrix $\mathbf{Z} \equiv [z_{ij}]$, Then, Leontief's input coefficients matrix $\mathbf{A} \equiv [a_{ij}] \geqq \mathbf{0}$ is constructed assuming proportionality of inputs z_{ij} and outputs x_j, $a_{ij} = \frac{z_{ij}}{x_j}$, for $i, j \in N$. Finally, Leontief's inverse $\boldsymbol{\Lambda} \equiv [\Lambda_{ij}]$, i.e., the matrix that maps direct into total quantities, is obtain as $\boldsymbol{\Lambda} \equiv (\mathbf{I} - \mathbf{A})^{-1}$.[B.1]

Unit labor costs (ω_j, Ω_j) **and unit profits** (π_j, Π_j)**.** The direct unit labor costs and unit profits for industry j for each country-year is obtained dividing, for each industry, the total labor compensation (LAB) and total capital compensation (CAP) by gross output (GO), respectively, $\omega_j = \frac{\mathrm{LAB}_j}{\mathrm{GO}_j}$ and $\pi_j = \frac{\mathrm{CAP}_j}{\mathrm{GO}_j}$. For each country-year we construct vectors $\boldsymbol{\omega}' \equiv [\omega_j]$ and $\boldsymbol{\pi}' \equiv [\pi_j]$. The vectors with the *total* unit labor costs and unit profits are obtain as $\boldsymbol{\omega}'\boldsymbol{\Lambda} \equiv \boldsymbol{\Omega}' \equiv [\Omega_j]$ and $\boldsymbol{\pi}'\boldsymbol{\Lambda} \equiv \boldsymbol{\Pi}' \equiv [\Pi_j]$, respectively.

Profit-wages ratios (ψ_j, Ψ_j)**.** Let $\hat{\mathbf{y}} = \mathrm{diag}\{y_1, \ldots, y_n\}$ for any vector $\mathbf{y}' \equiv [y_j]$. Then, for each country-year, the vectors with industries' direct and total profits-wages ratios are obtained as $\boldsymbol{\pi}'\hat{\boldsymbol{\omega}}^{-1} \equiv \boldsymbol{\psi}' \equiv [\psi_j] \equiv [\frac{\pi_j}{\omega_j}]$ and $\boldsymbol{\Pi}'\hat{\boldsymbol{\Omega}}^{-1} \equiv \boldsymbol{\Psi}' \equiv [\Psi_j] \equiv [\frac{\Pi_j}{\Omega_j}]$, respectively.

Capital intensities. The direct unit capital advanced in industry j for each country-year is obtained aggregating the value of intermediate inputs (II) and the nominal stock of capital ($\mathcal{K}$) and then dividing it by gross output (GO), $\kappa_j = \frac{\mathrm{II}_j + \mathcal{K}_j}{\mathrm{GO}_j}$. On the other hand, the direct unit labor in industry j for each country-year is obtained dividing the number of persons engaged (EMP) by gross output (GO), $l_j = \frac{\mathrm{EMP}_j}{\mathrm{GO}_j}$. For each country-year we construct vectors $\boldsymbol{\kappa}' \equiv [\kappa_j]$ and $\mathbf{l}' \equiv [l_j]$. The vectors with the total unit capital advanced and total unit labor are obtain as $\boldsymbol{\kappa}'\boldsymbol{\Lambda} \equiv \mathbf{K}' \equiv [K_j]$ and $\mathbf{l}'\boldsymbol{\Lambda} \equiv \mathbf{v}' \equiv [v_j]$, respectively. Then, for each country-year, the vector with industries' total capital intensities is obtained as $\frac{K_j}{v_j}$ for $j \in N$.

[B.1] In each case $(\mathbf{I} - \mathbf{A})^{-1} \geqslant \mathbf{0}$.

C Empirical distribution of capital intensities

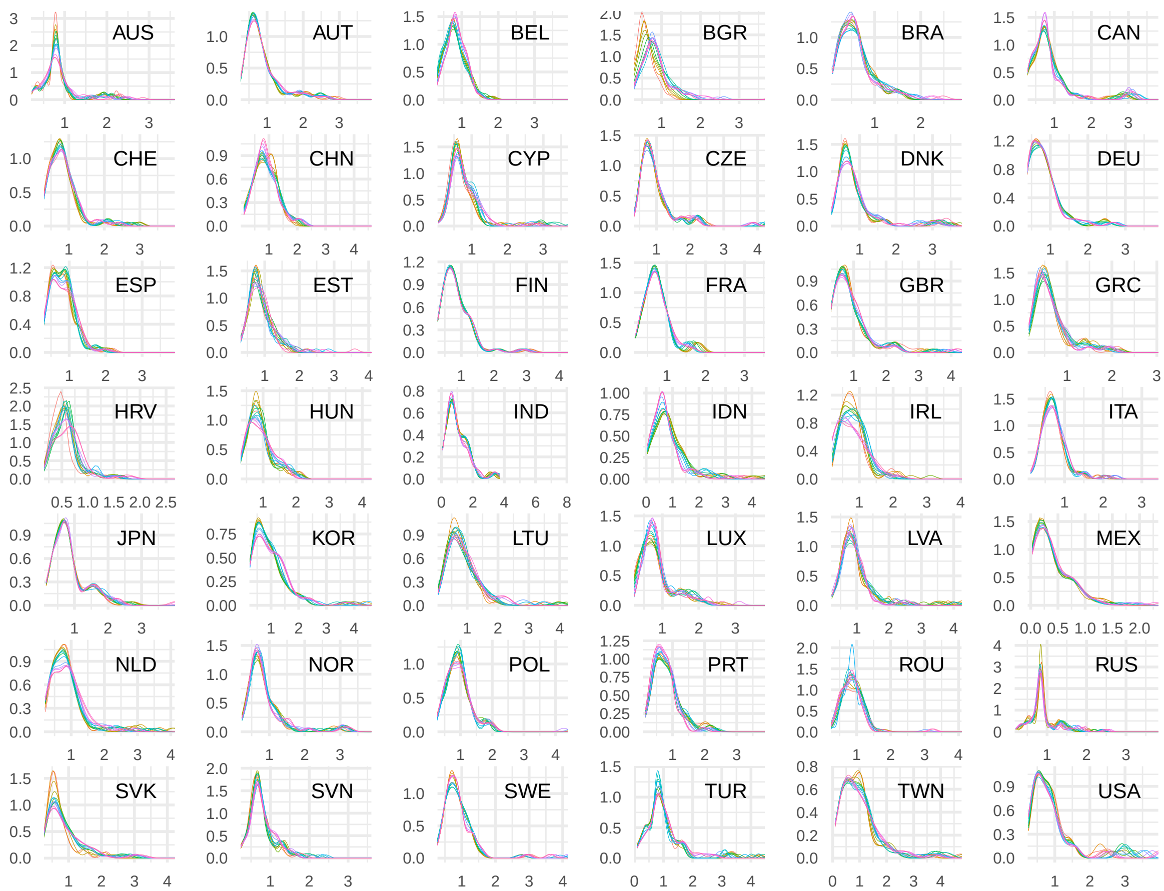

Figure C.1: Total capital intensities of industries within countries. *Source*: authors' calculations using the World Input-Output Database.

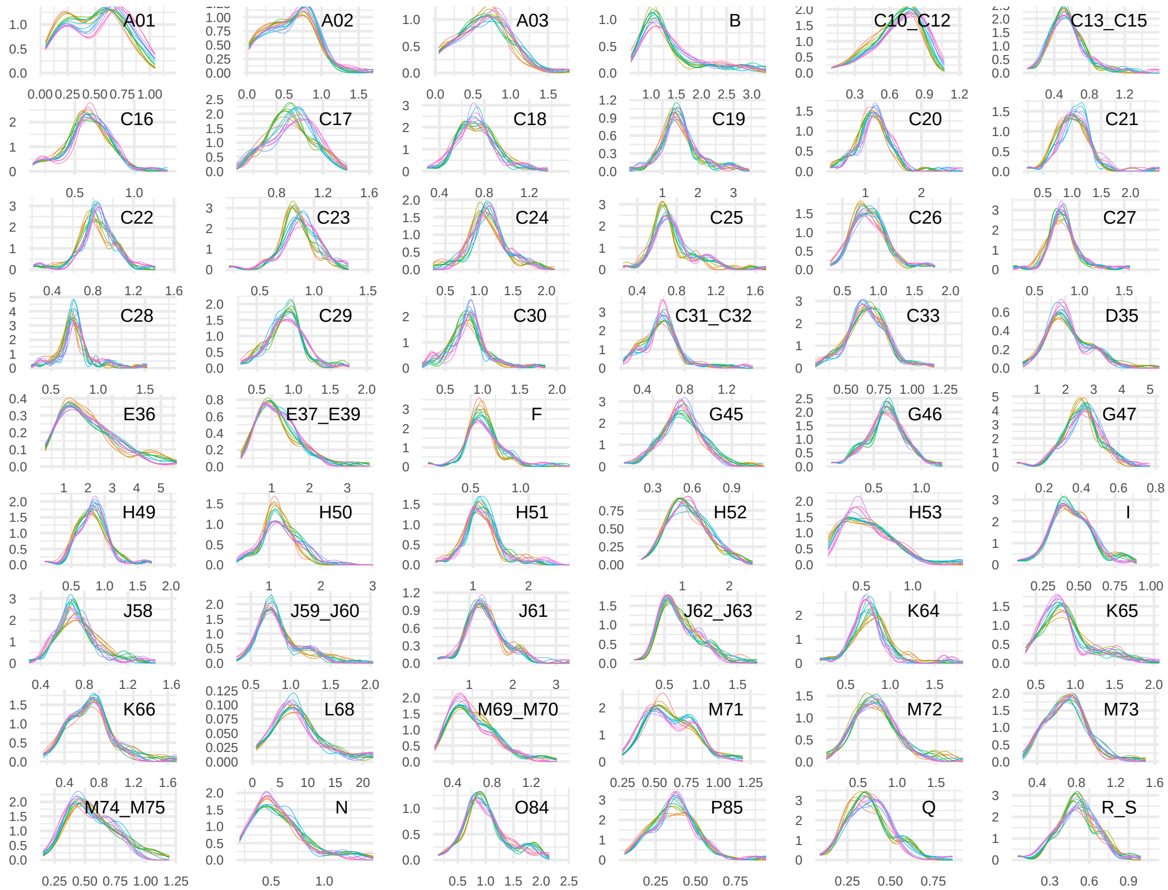

Figure C.2: Total capital intensities of industries across countries. *Source*: authors' calculations using the World Input-Output Database.

D Details on the Bayesian methodology for testing the Hypothesis of Ψ_j^A and Ψ_k^B mean-overlap

D.1 The Bayesian approach

The Bayesian approach to statistical inference defines probabilities as the theoretical likelihoods or "degrees of belief" about the frequencies with which outcomes are observed. This implies treating parameters as random variables and the data as fixed, which is the opposite approach to standard frequentist inference.

Drawing from basic probability theory, Bayes' rule provides a straightforward way of finding the *posterior* probability distribution $p(\theta|y)$ of a parameter θ given the known value of the data y by multiplying the *prior* probability of parameter θ and the conditional probability distribution $p(y|\theta)$, known as the likelihood function. The latter encapsulates the key distributional assumption on the data-generating process, and the former is our prior knowledge of the parameters. Dividing by the marginal distribution $p(y)$ we obtain the posterior density properly, but it suffices with the unnormalized posterior density $p(\theta|y) \propto p(\theta)p(y|\theta)$.

This framework offers substantial modeling flexibility, an intuitive interpretation of the uncertainty of the estimation, and, most importantly, a parsimonious way of exploiting prior information (McElreath, 2020). The posterior probability distribution is a weighted average of the likelihood and the prior, where the importance of the prior decreases substantially along with the uncertainty as more data becomes available. Therefore, the region populated by draws from the posterior distribution of any parameter charts the credible predictive interval where the mean of the actual distribution could be given the information contained in the original data.

D.2 Effect sizes of the different means

Following Kruschke (2013), we compute the effect size as $(\mu_1 - \mu_2)/\sqrt{(\sigma_1^2 + \sigma_2^2)/2}$. This is a simplification which, according to this author, introduces no relevant difference concerning the standard formula that accounts for the difference in the sample size of the groups, which in this case are almost equal in all cases.

D.3 The Gamma model for Ψ_j^α

The Gamma model consist on

$$\Psi_j|\alpha, \beta \sim \textbf{Gamma}(\alpha, \beta) \tag{D.1}$$

$$\alpha \sim \textbf{Normal}(0, 10) \tag{D.2}$$

$$\beta \sim \textbf{Normal}(0, 10) \tag{D.3}$$

Following the literature, we assign weakly informative priors to the parameters (α, β) that do not compromise the results but help model convergence (Gelman et al., 2013).

The full implementation of the Gamma model is done by posterior simulation using the Hamiltonian Monte Carlo (HMC) algorithm via *Stan* (Team, 2022) (for a more general introduction see Betancourt (2018)). The model runs 4 Markov chains with 3000 iterations each, of which 1000 are warm-up iterations. The model converges and shows no pathological behavior that could compromise the results of the estimation, e.g. more than 5% divergent transitions or bad Pareto k-factors. The scale reduction factor ($\hat{R}$) and the bulk Effective Sample Size (ESS) are below 1.01 and safely above the 400 samples recommended, respectively.

D.4 Table 1 for all countries, 2014: 95% highest density credibility bound of $\mu^\alpha - \mu^\beta$

	AUS	AUT	BEL	BGR	BRA	CAN	CHE	CHN	CYP	CZE	DEU	DNK	ESP	EST	FIN	FRA	GBR	GRC	HRV	HUN
AUT	0.03																			
BEL	0.2																			
BGR			0.13																	
BRA	0.36	0.55	0.68	0.37																
CAN			0.14		0.38															
CHE	0.41	0.15	0.01	0.31	0.83	0.34														
CHN		0.11	0.29		0.31		0.51													
CYP					0.46		0.13													
CZE		0.17	0.36		0.29		0.6		0.05											
DEU	0.1			0.03	0.6	0.05	0.08	0.18		0.24										
DNK			0.01		0.39		0.18													
ESP			0.11		0.36		0.29				0.02									
EST		0.14	0.32		0.27		0.54		0.03		0.21									
FIN	0.11			0.05	0.6	0.05	0.01	0.18		0.23			0.03	0.21						
FRA	0.28	0.05		0.2	0.73	0.22		0.36	0.04	0.42		0.1	0.19	0.39						
GBR	0.26	0.02		0.18	0.72	0.2		0.34	0.02	0.4		0.07	0.17	0.37						
GRC	0.33	0.51	0.64	0.35		0.36	0.78	0.28	0.44	0.27	0.57	0.37	0.34	0.26	0.57	0.69	0.68			
HRV	0.13			0.07	0.62	0.08		0.2		0.25			0.06	0.23				0.59		
HUN		0.1	0.28		0.33		0.51				0.17				0.17	0.36	0.34	0.31	0.19	
IDN	0.21	0.44	0.61	0.22		0.24	0.81	0.15	0.32	0.14	0.51	0.24	0.21	0.11	0.5	0.67	0.65		0.51	0.18
IND	0.56	0.8	0.99	0.55		0.57	1.24	0.5	0.65	0.51	0.88	0.54	0.53	0.45	0.85	1.03	1.02		0.84	0.54
IRL		0.04	0.19		0.23		0.36				0.1				0.11	0.26	0.23	0.22	0.14	
ITA			0.12		0.32		0.29				0.03				0.05	0.2	0.17	0.3	0.08	
JPN			0.13		0.31		0.31				0.05				0.06	0.21	0.19	0.29	0.08	
KOR					0.51		0.14	0.06		0.11				0.09		0.05	0.02	0.48		0.05
LTU	0.2	0.43	0.6	0.22		0.23	0.8	0.15	0.31	0.13	0.5	0.23	0.21	0.11	0.49	0.66	0.64		0.5	0.18
LUX	0.27	0.04		0.2	0.72	0.21		0.34	0.04	0.41		0.09	0.18	0.38				0.69		0.34
LVA		0.14	0.33		0.3		0.56		0.03		0.22				0.21	0.39	0.37	0.28	0.23	
MEX	1.81	1.97	2.09	1.8	0.99	1.82	2.23	1.78	1.86	1.78	2.01	1.78	1.78	1.75	2	2.12	2.11	0.9	1.99	1.81
MLT	0.07				0.58	0.01	0.14	0.15		0.21				0.18		0.03	0.01	0.54		0.14
NLD					0.51		0.17	0.06		0.11				0.09		0.07	0.05	0.48		0.05
NOR			0.01		0.45		0.19			0.03				0.01		0.1	0.07	0.43		
POL	0.16	0.41	0.6	0.18	0.06	0.19	0.83	0.1	0.28	0.09	0.49	0.19	0.16	0.06	0.47	0.65	0.63	0.04	0.48	0.14
PRT	0.03	0.25	0.39	0.05		0.07	0.55		0.16		0.31	0.09	0.06		0.31	0.45	0.43		0.33	
ROU	0.54	0.75	0.9	0.54		0.56	1.08	0.49	0.63	0.48	0.81	0.55	0.53	0.46	0.8	0.95	0.93		0.8	0.52
RUS	0.27	0.03		0.19	0.74	0.21		0.36	0.02	0.44		0.08	0.18	0.4				0.7		0.36
SVK	0.24	0.47	0.64	0.25		0.27	0.83	0.18	0.35	0.17	0.54	0.27	0.25	0.14	0.53	0.69	0.67		0.54	0.22
SVN	0.23			0.16	0.69	0.17		0.31		0.37		0.05	0.14	0.34				0.66		0.3
SWE		0.05	0.22		0.36		0.44				0.12				0.13	0.3	0.27	0.34	0.14	
TUR	1.17	1.35	1.49	1.16	0.35	1.19	1.64	1.13	1.24	1.13	1.41	1.16	1.15	1.1	1.39	1.52	1.51	0.28	1.38	1.16
TWN					0.5			0.04		0.08				0.07				0.5		0.04
USA		0.11	0.28		0.26		0.47		0.01		0.18				0.18	0.35	0.32	0.24	0.21	

Table D.1: 95% credibility bound of the pair-wise difference of posterior distributions of the mean parameters of the total profit-wage ratios for all countries ($\mu^{\alpha} - \mu^{\beta}$). Missing values indicate a possible overlap with a lower than 95% credibility interval for the null. *Source*: authors' calculations.

	IDN	IND	IRL	ITA	JPN	KOR	LTU	LUX	LVA	MEX	MLT	NLD	NOR	POL	PRT	ROU	RUS	SVK	SVN	SWE	TUR
IND	0.04																				
IRL	0.06	0.35																			
ITA	0.17	0.47																			
JPN	0.15	0.46																			
KOR	0.39	0.73																			
LTU		0.04	0.05	0.16	0.15	0.38															
LUX	0.65	1.02	0.25	0.19	0.21	0.04	0.65														
LVA	0.14	0.51				0.09	0.14	0.38													
MEX	1.43	1.25	1.64	1.73	1.72	1.92	1.42	2.11	1.78												
MLT	0.49	0.86	0.07		0.02		0.48	0.02	0.19	2.01											
NLD	0.39	0.74					0.39	0.06	0.09	1.93											
NOR	0.31	0.64					0.31	0.09	0.01	1.86											
POL		0.19		0.11	0.09	0.35		0.64	0.09	1.56	0.46	0.35	0.27								
PRT		0.05		0.01	0.01	0.21		0.44		1.39	0.28	0.2	0.14								
ROU	0.1		0.39	0.48	0.47	0.7	0.09	0.94	0.49	1.03	0.79	0.7	0.62	0.21	0.12						
RUS	0.68	1.08	0.25	0.18	0.2	0.02	0.68		0.41	2.14	0.01	0.05	0.08	0.68	0.45	0.97					
SVK			0.1	0.2	0.18	0.42		0.68	0.18	1.38	0.51	0.42	0.35			0.05	0.71				
SVN	0.62	0.99	0.21	0.16	0.16		0.61		0.34	2.09		0.03	0.05	0.61	0.41	0.91		0.65			
SWE	0.21	0.57					0.21	0.28		1.82	0.09			0.17	0.04	0.55	0.3	0.25	0.25		
TUR	0.76	0.56	1.01	1.1	1.09	1.3	0.77	1.51	1.14	0.24	1.39	1.31	1.24	0.89	0.76	0.36	1.54	0.72	1.48	1.19	
TWN	0.32	0.57	0.03				0.33		0.06	1.8				0.27	0.22	0.61		0.36			1.2
USA	0.09	0.42				0.07	0.09	0.33		1.71	0.15	0.06		0.03		0.43	0.35	0.12	0.3		1.07

Table D.2: 95% credibility bound of the pair-wise difference of posterior distributions of the mean parameters of the total profit-wage ratios for all countries ($\mu^{\alpha} - \mu^{\beta}$). Missing values indicate a possible overlap with a lower than 95% credibility interval for the null. *Source*: authors' calculations.