Revealing the Multiple Associations Among Enterprises: An Empirical Study of Chinese Real Estate Enterprises Using A "Capital-Industry" Multi-layer Network

Abstract: This paper introduces a novel approach for identifying the multiple associations among enterprises and their structural features based on multilayer network analysis. Taking Chinese real estate enterprises as a case study, this study constructs a "capital-industry" multilayer network model based on over 400 million records of investments and divestments among publicly listed real estate enterprises from 2005 to 2020, as well as data from China's input-output tables. This model is built upon the actual flows of capital and products, delineating the multifaceted relationships of investment, divestment, and production supply between real estate enterprises and firms in other sectors, while analyzing the structural evolution of these multiple associations. The findings offer new theoretical perspectives and tools for further application in areas such as risk spillover, behavioral decision-making, and asset pricing, contributing to the identification of risk contagion channels, exploration of decision-making mechanisms, and enhancement of price discovery processes.

Keywords: Multiple associations; Input-output connection; Capital association; Multilayer network; Network topology; Real estate enterprise

1 Introduction

Against the backdrop of the information technology revolution and economic globalization, enterprises have become increasingly interconnected through a multitude of interactions encompassing transactions, guarantees, shareholding, and upstream-downstream connections (Li, 2019; Li, 2021; Xu, 2022). As a result, enterprises are more likely to exert influence on each other through knowledge transmission, sentiment propagation, and risk contagion (Li et al., 2020; Liu et al., 2020; Liu et al., 2024). When an enterprise involved in these relationships experiences a crisis, other related enterprises may also be affected. Given the complexity of these interactions, it is of paramount importance to rigorously examine the multifaceted relationships between enterprises and their repercussions (Novoselova, 2022).

Complex networks have emerged as a prevalent approach for studying interconnectivity due to their ability to accurately reveal the intricate relationships among entities and markets through statistical characteristics. This method has yielded rich research findings in understanding correlations within financial markets, commodity markets, and real estate markets (Wang et al., 2023; Xu and Zhang, 2023; Gong et al., 2023; Xu et al., 2024). However, Existing studies have predominantly focused on singular connections between enterprise, lacking a unified framework for simultaneously studying multiple connections and considering interactions between different types of connections. In reality though, enterprises often exhibit concurrent multiple types of relationships that are operational at the same time (Jeude et al., 2019). Therefore, there is an increasing recognition in academia towards analyzing correlations based on multilayer networks compared to single-layer networks. Multilayer complex networks are more sensitive to risk identification, enabling earlier detection through the recognizing multiple correlations (Dai et al., 2023; Foglia et al., 2023), while also providing comprehensive information inaccessible by single-layer networks (Liu and Huang, 2022).

Hence, this study uses complex network analysis to characterize the intricate connections among real estate enterprises, which are key players in the market economy and have multifaceted relationships with firms in other industries (Chan et al., 2016). Through these connections, real estate enterprises also exert spillover effects on firms in other industries such as financial industries (Pais & Stork, 2011; Zhang et al., 2019; Chiang & Chen, 2022). In this context, investigating the complex and interconnected relationships among enterprises, with a focus on real estate enterprises as a case study, has significant implications for identifying channels of risk contagion, exploring decision-making mechanisms, and enhancing price discovery processes in the real estate market and its associated sectors.

This study contributes to the existing literature by introducing a novel methodology for assessing the interconnectedness of enterprises through multilayer network analysis. This approach holds great potential for applications in the fields of behavioral finance, asset pricing, and risk management within the economic and financial sectors. It facilitates the identification of channels for contagion risk, examination of decision-making processes, and improvement of price discovery mechanisms, providing valuable insights into pathways for transmitting risk and levels of exposure, thereby assisting both enterprises and regulatory authorities in maintaining stability within the financial system.

Our study is related to the works of Gong et al. (2022), Li et al. (2023) and Wang et al. (2023). Gong et al. (2022) constructed a multiplex social association network of institutional investors and calculated the clustering coefficient to characterize information interaction under multiple associations, and further verified its impact on stock price crash risk. While their work provides valuable reference for our research, we propose a more comprehensive analysis index of multilayer network topology to better analyze the complexity of relationships between market participants. Wang et al. (2023) introduced an interconnected multilayer network framework based on variance decomposition and block aggregation technique to quantify connectedness among global stock and foreign exchange markets, while Li et al. (2023) respectively used high-frequency TENET network and Granger-causality network to measure tail risk of stock price volatility and investor sentiment contagion, then constructing a two-layer network. The two latest studies by Chen et al. (2024) and Shen et al. (2024) also constructed multi-layer networks based on price correlation to analyze correlations between industries in the stock market as well as risk linkages between them. We distinguish our study by constructing a multi-layer association network according to actual factor correlations among market participants, introducing various multi-layer network topology indicators to identify the multiple associations among market participants, which provide a micro behavioral foundation for the network established based on correlation analysis.

2 Method for Identifying Multiple Associations among Enterprises Based on Multilayer Networks

2.1 Construction of the "Capital-Industry" Multilayer Network

The "Capital-Industry" multilayer network is constructed by considering focus enterprises and their associated companies as nodes, while investment, investment-receiving, and input-output associations between enterprises serve as edges, thereby establishing a multiplex network. The specific methodology for constructing this network is outlined below.

2.1.1 Capital Association Network (hereinafter referred to as SH layer)

To construct the capital association network, it is necessary to gather information on the top 10 shareholders of focus enterprises and the outward investment situations of those enterprises. Subsequently, we'll utilize the focus enterprises and their associated companies as nodes in the network. If a shareholding or investment relationship exists between a focus enterprise and any other enterprise, an edge will be established between them in the network; otherwise, no edge will be present. It should be noted that due to undisclosed proportions of outward investment shareholding by focus enterprises and unavailability of data, the resulting capital association network will be a directed unweighted network."

2.1.2 Industry Association Network (hereinafter referred to as IO layer)

The nodes in the industry association network align with those in the capital association network. The input-output table is utilized to depict the upstream and downstream supply relationships between focus enterprises and their associated companies. In order to establish inter-enterprise industry associations, companies within the network are initially categorized based on production sectors outlined in the input-output table. If one industry's output relies on another industry, it is deemed that a production association exists between these two industries, resulting in an edge connecting their respective nodes. To further capture the strength of industry associations within the network, directed and weighted connections are constructed using direct consumption coefficients derived from sector-specific data provided by the input-output table. The calculation formula for the direct consumption coefficient a_{ij} is as follows:

$$a_{ij} = \frac{x_{ij}}{x_j} \tag{1}$$

where a_{ij} denotes the value of department j directly consuming products from department i; X_{ij}represents the quantity or value of goods from department i directly consumed in the production process of department j; and X_j represents the total input of department j. It should be noted that the direct consumption coefficient a_{ij} falls within the range of $0 \leq a_{ij} < 1$, where a larger value indicates a stronger industry association between enterprise *i* and enterprise *j*.

The "Capital-Industry" multilayer network constructed according to the aforementioned method is illustrated in Figure 2.1 below. Figure 2.1 illustrates a multilayer network consisting of 10 enterprises, with node C representing one of the enterprises in the network. To enhance clarity, we categorize the capital association network into two subnetworks: the investment network of focus enterprises and the shareholding network. For instance, within the industry association network, an arrow from node C2 to C6 indicates direct consumption from industry 2 to industry 6; within the investment association network, an arrow from node C1 to C4 represents enterprise 1's investment in enterprise 2; and within the shareholding association network, an arrow from node C6 to C3 signifies the investment made by enterprise 6 in enterprise.

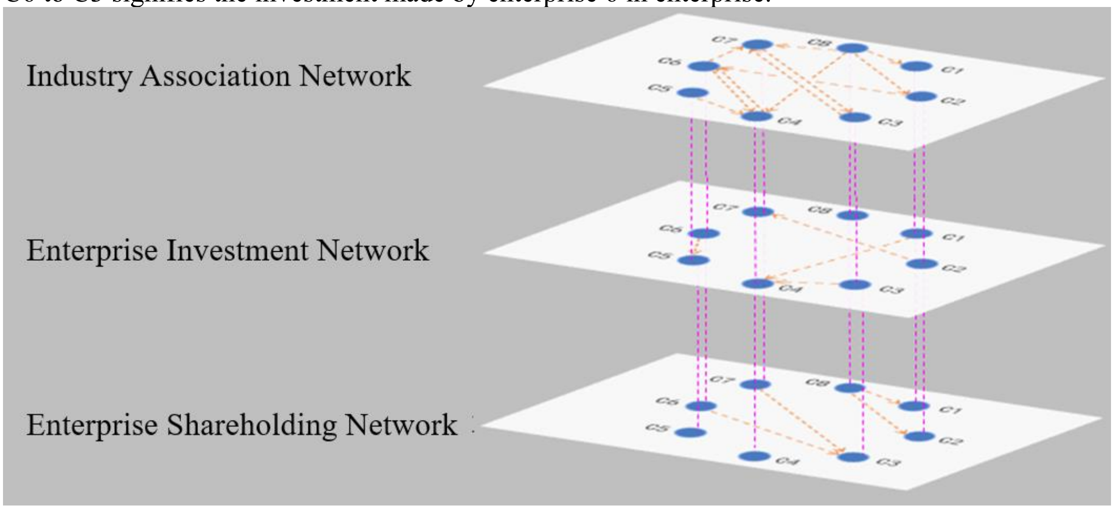


Figure 2.1 Legend of the "Capital-Industry" Multilayer Network

2.2 Multilayer Network Structure Characterization and Multiple Association Identification

2.2.1 Characteristics of enterprises: Node properties in multilayer networks

A. External Influence of enterprises: Node Out Degree (OD_i) and Node connect-out strength

Node Out Degree (OD_i). The OD of a node i in a directed network is defined as the number of edges that are connected to it. In a system composed of *N* nodes and *M* unweighted layers, each single-layer network has an adjacency matrix $A^{[\alpha]} = \{a_{ij}^{[\alpha]}\}$. When there is an out edge between node *i* and node *j*, $a_{ij}^{[\alpha]}$=1 and vice versa is 0. Therefore, the out degree of the single layer network can be calculated as follows:

$$OD_i^{[\alpha]} = \sum_j a_{ij}^{[\alpha]} \tag{2}$$

The out degree of node i in the multilayer network with M layers can be represented as a vector, where α denotes the layer of the node and $0 \leq OD_i^{[\alpha]} \leq N-1, \forall i, \forall \alpha$.

$$\overrightarrow{OD_i} = \{OD_i^{[1]}, \dots, OD_i^{[M]}\}, i = 1, \dots, N \tag{3}$$

In order to enhance the accuracy of measuring the out degree of the integrated multilayer network, we employ a compression technique on the multilayer network, and the aggregated topological adjacency matrix $A = \{a_{ij}\}$ is obtained. Consequently, the out degree of node *i* within the *M*-layer network is determined as follows:

$$K_i = \sum_j a_{ij} \tag{4}$$

When discussing the value of a_{ij}, it is necessary to separately consider whether the multilayer network incorporates a single-layer network with edge weights. If there are no edge weights assigned to any outgoing edges connecting node i and node j in any single-layer network within the multilayer structure, the following condition holds true: a_{ij} = 1 if and only if $a_{ij}^{[\alpha]}$ = 1 $\exists\alpha$, conversely, a_{ij}= 0. However, if all connected outgoing edges from node i to node j have assigned weights, then the condition for a_{ij} = 1 changes to a_{ij} = 1 if and only if $w_{ij}^{[\alpha]}$ > 0 $\exists\alpha$, otherwise, a_{ij} = 0.

Node connect-out strength. The node connect-out strength, as introduced by Battiston et al. (2014), is employed to extend the node out-degree to networks with edge weights. Specifically, it represents the sum of the edge weights associated with all connect-out degrees of node *i* in the α-layer. Mathematically, it can be expressed as:

$$s_i^{[\alpha]} = \sum_j w_{ij}^{[\alpha]} \tag{5}$$

Similarly, the node-connected out strength of node i in a multilayer network with M layers can be represented as the vector:

$$\vec{s_i} = \{s_i^{[1]}, \dots, s_i^{[M]}\},\ i = 1, \dots, N \tag{6}$$

B. System importance of enterprises: Node vector centrality

In the analysis of complex networks, the centrality analysis of nodes remains a significant research question in extending various measures of centrality, such as node degree, proximity centrality, and intermediate centrality from single-layer to multi-layer networks.

The analysis of the characteristic vector centrality within multilayer networks is a key focus in this study, representing an extension of the notion of degree centrality. Within a single-layer network context, the characteristic vector centrality of node *i* is delineated as the *i*-th element of the characteristic vector associated with the principal eigenvalue of the network's adjacency matrix. For multiplexed networks, we can calculate eigenvector centrality at each layer. Let the eigenvector centrality of node i at layer α be $E_i^{[\alpha]}$, then the eigenvector centrality of node i in layer M multiplexed network becomes a vector:

$$E_i = \left\{E_i^{[1]}, \dots, E_i^{[M]}\right\} \tag{7}$$

2.2.2 Characteristics of inter-enterprise associations: Interlayer correlations in multi-layer networks

A. Inter-enterprise association strength: Edge overlapping degree

With reference to the calculation methods in existing literature, the edge overlapping degree between nodes *i-j* is defined as the frequency of its occurrence in an M-layer multilayer network. Firstly, we consider the edge overlapping degree of the unweighted network, which can be calculated as follows:

$$o_{ij} = \sum_\alpha a_{ij}^{[\alpha]}\ ,\ 0 \le o_{ij} \le M, \forall i, j. \tag{8}$$

As a result, the aggregated overlapping adjacency matrix O (aggregated overlapping adjacency matrix) can be obtained as $\{o_{ij}\}$ according to the above definition.

By deriving from the edge overlapping degree of the edge-weightless network, the edge overlapping degree of the edge-weighted network can be obtained:

$$o_{ij}^w = \sum_\alpha w_{ij}^{[\alpha]} \tag{9}$$

The aggregated overlapping neighbor matrix with weights is $0^w\{o_{ij}\}$.

B. Diversity of association: Balance of nodes degree distribution across all layers

The analysis of node connectedness alone fails to give a comprehensive characterization of the degree distribution characteristics for node *i* in a multi-layer network. Therefore, referring to the existing literature, we employ the multiplex participation coefficient to describe the degree distribution of node *i* across all layers and quantify its level of participation within the network. The multiple participation coefficient P_i of node *i* is calculated as follows:

$$P_i = \frac{M}{M-1}\left[1 - \sum_{\alpha=1}^{M}\left(\frac{k_i^{[\alpha]}}{o_i}\right)^2\right] \tag{10}$$

In multi-layer networks, the value of P_i should be constrained within the range [0,1]. When all outgoing edges from node i reside within the same layer, P_i is assigned a value of 0; conversely, when node i exhibits an equal number of edges across each layer in a multilayer network, P_i is set to 1. In essence, a higher the value of P_i indicates a uniform the distribution of node *i* 's participation throughout the multi-layer network.

C. Differences in association: Degree distribution disparities across different layers

Degree distribution is a fundamental characteristic in the analysis of single-layer networks, while in multi-layer networks, the degree distribution varies across different layers. A node with high connectivity in one layer may exhibit limited or even isolated connectivity in another layer. Hence, it is crucial to assess the distribution of node connectivity across different layers.

In this study, we conduct an analysis on the degree distribution of multi-layer networks by calculating the aggregate topologies k_i and $k^{[\alpha]}$ of nodes in $\alpha \in \{IO, SH\}$ layer. Subsequently, the nodes are sorted according to their order of aggregate topologies. Furthermore, in order to better quantify this correlation, we compute the Kendall correlation coefficient τ_k to assess the similarity of the two ranking sequences of data *X* and *Y*. A value of 1 for τ_k *(X, Y)* indicates identical rankings, while -1 signifies completely opposite rankings; If the rankings of the two sequences are completely opposite, a value of 0 suggests complete independence.

D. Correlation of association: Correlation between different layers of multi-layer networks

After proposing certain some measures of the role of an individual node in a multi-layer network, we now consider a slightly more intricate variable, namely, the conditional probability of edge coexistence between nodes in both the α layer and the α' layer. This refers to the likelihood of an edge existing in the α' layer given its existence in the α layer, which can be expressed as follows.

$$P\left(a_{ij}^{[\alpha']} | a_{ij}^{[\alpha]}\right) = \frac{\sum_{ij} a_{ij}^{[\alpha']} a_{ij}^{[\alpha]}}{\sum_{ij} a_{ij}^{[\alpha]}} \tag{11}$$

where the denominator represents the number of connected edges from the α layer, and the numerator denotes the count of connected edges that also exist in the α' layer.

3 Identification and Characteristic Analysis of Multiple associations among Chinese Real Estate Enterprises

3.1 Data Source and Preprocessing

To comprehensively cover the major cycles of the Chinese real estate industry and key regulatory nodes in the real estate market, the study period is set between 2005 and 2020. The data required for constructing the input-output network are sourced from China's input-output tables spanning this period. For the shareholding network, our data selection includes information on the top ten shareholders and external shareholding data of 119 real estate enterprises in the revised 2021 edition of the Shenwan Hongyuan Industry Classification, which encompasses a total of 17,170 companies across 40 industries, resulting in a comprehensive dataset comprising a total of 474,181,518 data points. We obtained top ten shareholder data of real estate enterprises from the CSMAR database, while acquiring external shareholding data and industry classifications for all enterprises through web scraping using web crawlers from "Tianyancha".

After obtaining the raw data, it must undergo processing to generate computer-recognizable adjacency matrices, subsequently enabling the construction of complex networks for analysis purposes. Given the vast number of enterprises involved, resulting in excessively large adjacency matrices, HPC (High-Performance Computing) platforms are employed for data processing in 2018 and 2020. Ultimately, a total of 16 adjacency matrices representing *IO* layers and *SH* layers from 2005 to 2020 are derived.

3.2 Identification of Multiple associations among Enterprises

3.2.1 Basic characteristics of the "Capital-Industry" Multilayer Network

The basic topological indicators for eight sets of multilayer networks spanning from 2005 to 2020 are computed. The primary multilayer network *M* in this study comprises industry association IO layers and capital association *SH* layers. It is evident from the out-degree indicator *OD* in Table 3.1 that almost every node in the *IO* layer has outgoing connections to other nodes, indicating a high level of interconnectivity among enterprises. Over time, the number of nodes in the "Capital-Industry" multilayer network has been increasing annually. This trend may be attributed to the expanding investment chains of real estate enterprises as the Chinese real estate market evolves. In 2005, most real estate enterprises had only one layer of external investment, indicating that subsidiaries of real estate enterprises did not engage in further investment activities. However, by

2020, the number of layers of external investment for some real estate enterprises had reached as high as 10 layers. The growth in investment chains leads to an expansion in the number of nodes within the network.

Table 3.1 Basic Characteristics of Multilayer Networks from 2005 to 2020

Year	*Layer*	*Symbol*	*N*	*OD*	*S*	*O*	O^w
2005	*Multiplex*	*M*	1072	1109727	/	1110743	25869.94
	Industry Association	*IO*	1072	1109705	24830.94	/	/
	Capital Association	*SH*	1072	1039	/	/	/
2007	*Multiplex*	*M*	1200	1428873	/	1430093	35581.42
	Industry Association	*IO*	1200	1428872	34360.42	/	/
	Capital Association	*SH*	1200	1221	/	/	/
2010	*Multiplex*	*M*	1572	2471104	/	2472749	335951.36
	Industry Association	*IO*	1572	2471104	334306.36	/	/
	Capital Association	*SH*	1572	1645	/	/	/
2012	*Multiplex*	*M*	1964	3838029	/	3840140	146707.38
	Industry Association	*IO*	1964	3838025	2074222.38	/	/
	Capital Association	*SH*	1964	2115	/	/	/
2015	*Multiplex*	*M*	1514	2276953	/	2278654	109487.68
	Industry Association	*IO*	1514	2276950	107783.68	/	/
	Capital Association	*SH*	1514	1704	/	/	/
2017	*Multiplex*	*M*	3630	2276953	/	2278654	109487.68
	Industry Association	*IO*	3630	2276950	107783.68	/	/
	Capital Association	*SH*	3630	1704	/	/	/
2018	*Multiplex*	*M*	11179	137232960	/	137245415	5609913.80
	Industry Association	*IO*	11179	137232828	5597326.80	/	/
	Capital Association	*SH*	11179	12587	/	/	/
2020	*Multiplex*	*M*	17170	290638671	/	290656525	12342739.6
	Industry Association	*IO*	17170	290638447	12324661.6	/	/
	Capital Association	*SH*	17170	18078	/	/	/

Notes. Table 3.1 illustrates the basic topological indicators for eight sets of multilayer networks spanning from 2005 to 2020. In the table, N represents the number of nodes in the network, S represents the strength of the nodes in the network, OD represents the out-strength of the nodes, O represents the edge overlapping degree of the network, and O^w denotes the edge overlapping degree of the edge-weighted network.

3.2.2 System importance of enterprises in the "Capital-Industry" Multilayer Network

After computing the centrality measures of the single-layer networks, further calculations were conducted to determine the centrality of the aggregated topology and the aggregated overlapping networks. The results are denoted as $E_i(A)$ and $E_i(O)$ respectively.

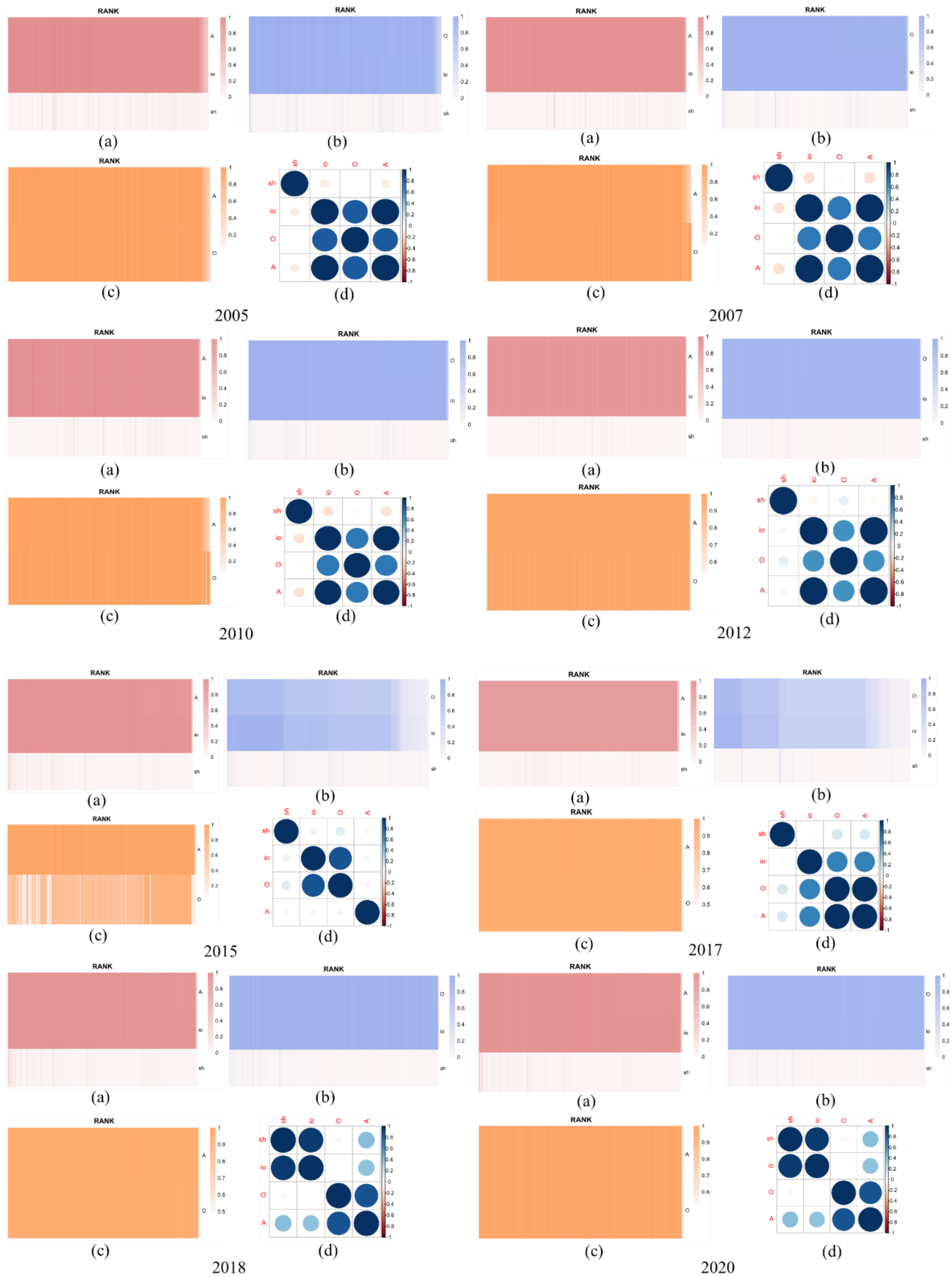


Figure 3.1 Analysis of Eigenvector Centrality in the Multilayer Network

Notes. In Figure 3.4(a), 3.4(b), and 3.4(c), the eigenvector centrality calculated on each layer is compared with the eigenvector centrality of the aggregated topology and aggregated overlapping network. The Kendall correlation coefficients of these centrality pairs are displayed in Figure 3.4(d) in the form of a heatmap. (a) The top row of the panel shows with a color code the eigenvector centrality $E_i(A)$ of each node in the aggregated topological network, from the largest (darkest, left most to the smallest (brightest, rightmost). Keeping fixed the ranking induced by $E_i(A)$, in the other two rows we report, respectively, the eigenvector centrality in the industry layer $E_i^{[IO]}$, and capital layer $E_i^{[SH]}$. (b) Similar to panel (a) but here the nodes are ranked according to their eigenvector centrality computed on the aggregated overlapping network $E_i(O)$. (c) Comparison of the rankings of eigenvector centrality computed on the aggregated topological network and on the aggregated overlapping network, respectively, $E_i(A)$ and $E_i(O)$. (d) The heat map shows the nonparametric correlation between the rankings induces by the different centralities.

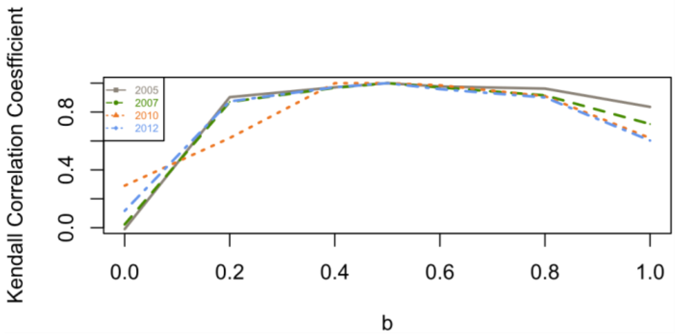
From Figure 3.1, it is evident that prior to 2015, for a significant portion of the nodes, the Kendall correlation between the eigenvector centrality of the SH layer and the centrality of the aggregated network is less than 0. This suggests that companies with high centrality in the shareholding network tend to have lower centrality in the industry association network and the overall aggregated topology network. Consequently, it can be inferred that before 2015, companies with more industry associations were less inclined to establish shareholding relationships with other companies. However, after 2015, a significant positive correlation is observed between the centrality of nodes in the SH layer and IO layer, indicating that companies positioned centrally in industry associations began to leverage their complex industry relationships for investments and rapidly became central nodes in the shareholding network.

An analysis of the significant events associated with this change reveals that in 2015, China experienced one of the most severe stock market crashes in its history, leading to a substantial economic downturn. In conjunction with the earlier discussed analysis on corporate diversification, networked companies may opt to hold shares in both upstream and downstream companies to reduce transaction costs, acquire additional supply chain resources, and overcome information dissemination barriers for enhanced development prospects. Therefore, companies with greater industry associations are more likely to invest in and retain shares within related industries, leading to a significant positive correlation in the centrality levels of nodes across the IO and SH layers.

To simultaneously evaluate the role of each layer's multi-center in a two-layer network, we adopt a recently proposed methodology. Given a two-layer network and its corresponding adjacency matrices $A^{[1]}$ and $A^{[2]}$, the following adjacency matrix is constructed:

$$M(b) = bA^{[1]} + (1-b)A^{[2]} \tag{12}$$

In this expression, $M(b)$ represents the convex combination of $A^{[1]}$ and $A^{[2]}$, where b is a parameter in the range [0,1]. We refer to such a matrix as a multi-adjacency matrix, which plays a crucial role in capturing the multiplex structure. It's important to note that the parameter b determines the relative contribution of each layer to the multiplex structure. When b = 0 (or b = 1), the duplex multi-adjacency matrix reduces to $A^{[2]}$ (or $A^{[1]}$). Setting b = 0.5 serves as a baseline scenario where both layers are given equal weight.



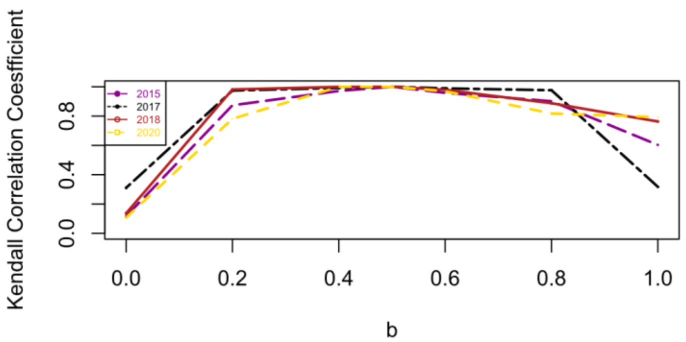


Figure 3.2 depicts the Kendall correlation coefficients between the centrality of the multi-layer

adjacency matrix and the aggregate overlap network.

Notes. For the multiplex network we plot the Kendall correlation coefficient τ_k between the eigenvector centrality of the benchmark case (*b*=0.5, i.e., equal weights on both layers) and the generic case of Eq. (12).

The specific procedure follows these steps: first, compute the centrality of the network based on the baseline scenario b = 0.5 using the given formula; subsequently, calculate the centrality of the network constructed from the general multi-adjacency matrix *M* for various values of *b*, and assess the Kendall correlation coefficient τ_k between the centrality obtained at $b = \bar{b}$ and the baseline scenario at *b* = 0.5. It's worth noting that *b* = 0.5 is chosen as the baseline scenario because when b = 0.5, M(*b*=0.5) $= \frac{O}{2}$, where O is the aggregate overlap network, thus *M* is directly proportional to the aggregate overlap network *O*. The resulting line chart of b versus τ_k is presented in Figure 3.2.

The construction method of the multi-layer adjacency matrix allows us to deduce from Figure 3.5 crucial information about the curve's slope, symmetry, and the Kendall correlation coefficients in the extreme cases where *b*=0 and *b*=1. These factors can collectively demonstrate the strength of the centrality between the IO layer and the SH layer in the multi-layer network.

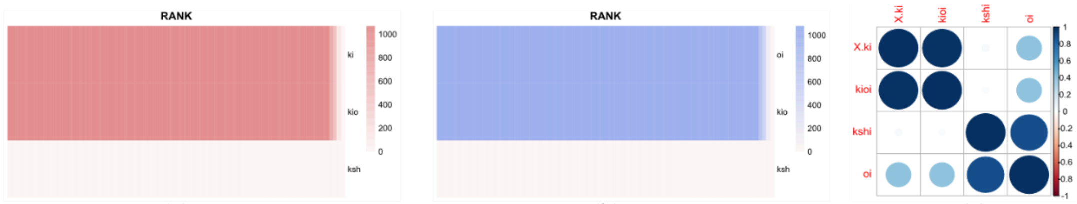
In Figure 3.5, when b = 0.5, the peak value ofτ_k for all 8 curves is observed to be 1. By comparing the eight curves from 2005 to 2020, it can be inferred that the IO and SH layers consistently demonstrate stable performance in determining centrality within the multi-layer network. This observation indicates that the influence of inter-enterprise industry associations on network structure and node importance persists throughout the entire time period, with the IO layer having a stronger effect over these eight years. Furthermore, the curves corresponding to IO-SH are asymmetric, suggesting that the IO layer dominates the SH layer in determining node centrality. Within the ranges of 0 < b < 0.5 and 0.5 < b < 1, similar trends are observed across all eight curves: steeper slopes are evident in the range of 0 < b < 0.5, while a more gradual trend is observed in the range of 0.5 < b < 1. This finding highlights that centrality is predominantly influenced by the IO layer rather than by the SH layer.

The empirical findings above suggest that the centrality of enterprises in this multi-layer network is primarily determined by their centrality within the industry association network, rather than solely by their capital connections. Consequently, when assessing the risk contagion of a specific firm, it is imperative to consider not only its capital associations but also the influence of its industry affiliation.

By comparing the two graphs in Figure 3.2, it is evident that, with 2015 as the dividing line, the amplitude of the upward trend on the left side of b = 0.5 during 2005-2007 is generally smaller than that during 2015-2020, indicating a heightened significance of industry associations in the multi-layer network after 2015. This finding supports previous speculation that companies were compelled to adopt related diversification strategies following the impact suffered in 2015.

3.2.3 Differences in association in the "Capital-Industry" Multilayer Network

Figure 3.3(a) sorts the out-degrees k_i of nodes in the aggregated topological network against the out-degrees $k_i^{[\alpha]}$ of nodes in the industry association *IO* layer and the capital association *SH* layer. Figure 3.3(b) presents the sorting of the out-degrees of nodes in the industry association *IO* layer and the capital association *SH* layer based on heir corresponding out-degrees $k_i^{[\alpha]}$ of nodes in the aggregated overlapping network, obtained by sorting nodes according to their node overlapping degree o_i. Figure 3.3(c) presents the Kendall correlation coefficient between k_i, o_i, and $k_i^{[\alpha]}$ in the form of a heatmap.

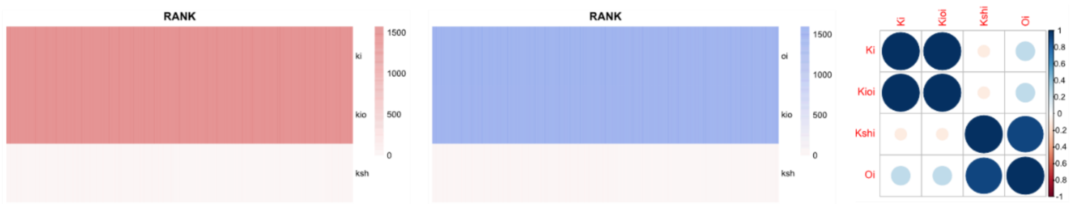


(a) (b) (c)

2005

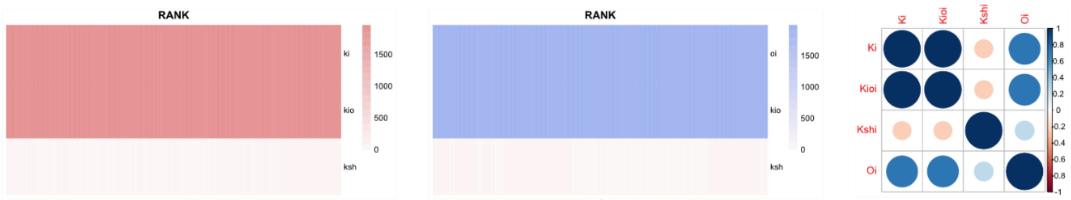
(a) (b) (c)

2007



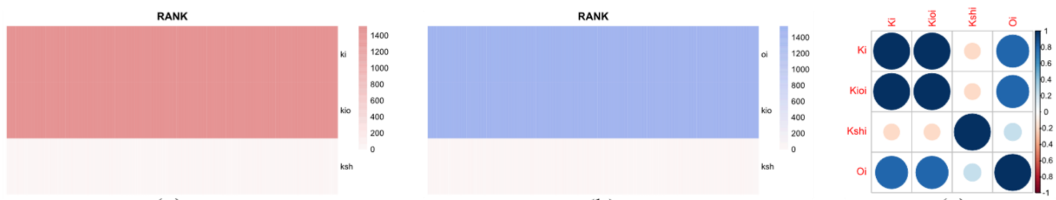
(a) (b) (c)

2010



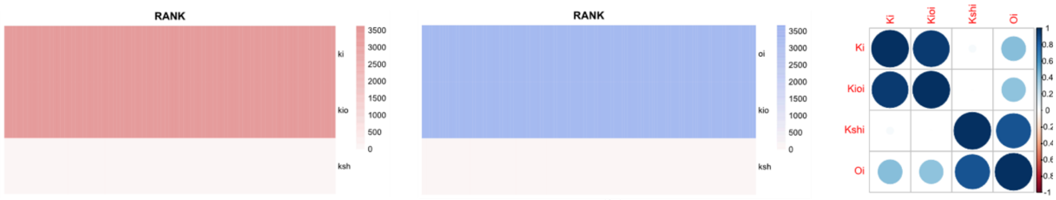
(a) (b) (c)

2012



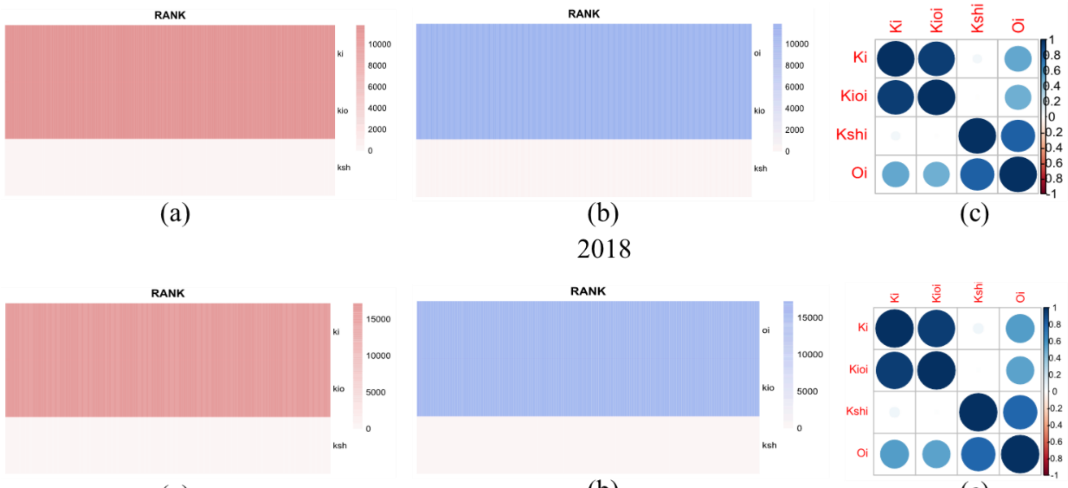
(a) (b) (c)

2015



(a) (b) (c)

2017



(a) (b) (c)
2018

(a) (b) (c)
2020

Figure 3.3 Out-Degree Analysis of the Multilayer Network (2005-2020)

Notes. (a) Keeping fixed the ranking induced by k_i, in the other three rows we report, respectively, the degree in the industry layer $k_i^{[IO]}$, noted in the figure as kio, and capital layer $k_i^{[SH]}$, noted in the figure as ksh. (b) Same as panel (a) but in the first row nodes are ranked according to their overlapping degree o_i. (c) The heat map represents the Kendall τ correlation coefficient among $k_i, o_i, k_i^{[IO]}$ and $k_i^{[SH]}$.

Prior to 2015, a significant correlation was observed between the out-degree $k_i^{[IO]}$ in the industry association network, the out-degree k_i in the aggregated topological network, and the out-degree o_i in the aggregated overlapping network. Hence, it can be inferred that industry associations play a dominant role in influencing the topological structure of the aggregated network within the multilayer network of real estate enterprises.

Before 2015, the Kendall correlation coefficient between the out-degree of nodes in the capital association network *SH* and the out-degree in both the industry association *IO* network and the aggregated overlapping network was below -0.4. However, after 2015, the Kendall correlation coefficient was almost 0, indicating no significant correlation between them. Thus, in this multilayer network, enterprises with stronger industry associations do not necessarily engage in extensive shareholding and investment activities.

3.2.4 Diversity of association in the "Capital-Industry" Multilayer Network

Figure 3.4(a) presents a scatter plot sorted from highest to lowest values of P_i in the *IO-SH* multilayer network, with a red dashed line representing the mean value of P_i. Figure 3.4(b) illustrates the z-scores of o_i by plotting the multi-participation coefficient P_i of each node *i* against its total overlapping degree o_i.

Due to the importance of node overlap in terms of the overall significance of event edges associated with the node, and the multi-participation coefficient providing information about the distribution of event edges across layers, an attempt is made to classify nodes in a multilayer network by simultaneously examining the multi-participation coefficient and overlapping degree. Referring to the definition in relevant literature regarding the multi-participation coefficient, all nodes are categorized into three classes based on the value of P_i: if $0 < P_i < \frac{1}{3}$, the node is referred to as a concentration node; if $\frac{1}{3} < P_i < \frac{2}{3}$, the node is referred to as a hybrid node; and if $P_i > \frac{2}{3}$, the node is referred to as a true multiplexing node. Furthermore, considering the Z-score of the overlapping degree o_i to compare different-sized multiplex networks:

$$Z(o_i) = \frac{o_i - <o>}{\sigma_o} \tag{11}$$

According to the literature, if $Z(o_i) > 2$, a node is considered to play the role of a central node in the network; otherwise, it is regarded as an ordinary node.

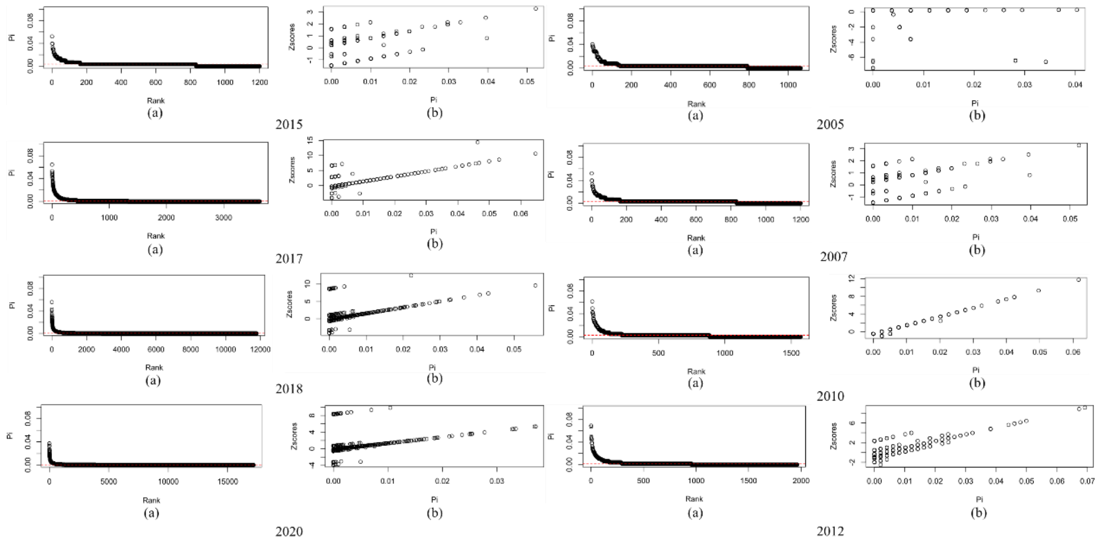


Figure 3.4 Node Multi-participation Coefficients in the Multilayer Network

Notes. (a) Rank distribution of the participation coefficient P_i for the multilayer network. The average value around P = 0.008 is shown as a horizontal red line. (b)For each node i, the multiplex participation coefficient P_i versus the Z score of the total overlapping degree $Z(o_i)$. By combining the two, we can determine that even if two nodes have exactly the same value of $Z(o_i)$, their levels of participation are not identical in the multilayer network.

Therefore, based on P_i and $Z(o_i)$, Figure 3.4(b) can be divided into six parts. However, it is evident from Figure 3.4 that most values of P_i in the *IO-SH* multilayer network are concentrated within the interval [0, 0.08], indicating that the majority of nodes i in the multilayer network are concentration nodes, and the out-going edges of the nodes are concentrated in the *IO* layer network. This observation aligns with the conclusion drawn from Figure 3.1. Nevertheless, a notable disparity is observed in Figure 3.4(b) for the year 2005: The Z-scores in 2005 range between [-6,0], suggesting that all nodes in this specific year are ordinary nodes in the network. This finding implies that compared to other years, the importance of enterprises within the network was relatively evenly distributed in 2005.

3.2.5 Correlation of association in the "Capital-Industry" Multilayer Network

To further explore the relationship between industry associations and capital associations among enterprises, Eq. (11). is used to estimate the edge overlap probability in the multilayer network.

Figure 3.5 depicts the heatmap of the conditional probability $P\left(a_{ij}^{[\alpha']}|a_{ij}^{[\alpha]}\right)$ for the network. For instance, the color-coded first row represents the probability of out-going edges present in the SH layer also appearing in both the *IO* layer and the *SH* layer. It is evident from Figure 3.5 that there is minimal disparity in the edge overlap probability between 2005 and 2020, and the probability of edges appearing in the SH layer simultaneously appearing in the *IO* layer is greater than the probability of out-going edges appearing in the *IO* layer and simultaneously appearing in the SH layer. Looking at the specific data, between 2005 and 2020, there is an average probability of 99% that out-going edges appearing in the SH layer also appear in the *IO* layer. This outcome suggests that the industry relationships among enterprises have a more substantial influence on their connections and collaborations in terms of shareholding and investment, within the multilayer network of real estate enterprises. This finding aligns with the research of Shi et al. (2021), indicating that increasing the shareholding of upstream enterprises in downstream enterprises can effectively reduce market prices and improve operational performance.

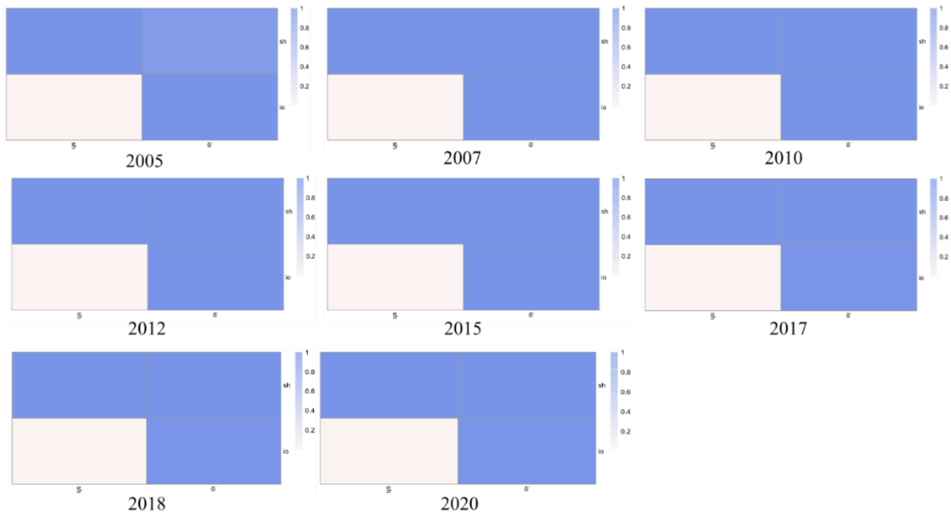


Figure 3.5 Edge Overlapping probability in the Multilayer Network

Notes. For each layer α, we show in the color map the fraction of edges which is also present in each other layer a'.

It is evident from Table 3.2 that real estate companies predominantly hold shares in industries situated upstream or downstream of the real estate production chain. Therefore, this analysis reaffirms that real estate companies adopt a diversified investment strategy based on relevant analysis when making shareholdings, specifically targeting enterprises within their upstream or downstream industry chains.

Table 3.2 Top 5 Industries of Enterprises Holding Shares in Real Estate Enterprises, 2005-2020

	1	2	3	4	5
2005	Real Estate	Leasing and Business Services	Accommodation and Food Services	Finance	Construction
2007	Leasing and Business Services	Real Estate	Accommodation and Food Services	Construction	Finance
2010	Construction	Leasing and Business Services	Real Estate	Finance	Accommodation and Food Services
2012	Real Estate	Leasing and Business Services	Accommodation and Food Services	Finance	Construction
2015	Construction	Leasing and Business Services	Accommodation and Food Services	Finance	Real Estate
2017	Real Estate	Construction	Finance	Leasing and Business Services	Accommodation and Food Services
2018	Real Estate	Leasing and Business Services	Construction	Finance	Accommodation and Food Services
2020	Real Estate	Leasing and Business Services	Accommodation and Food Services	Finance	Construction

3.3 Robustness Analysis and Discussion

To further elucidate the significance of our study, we employ the recent case of *Evergrande Group*, which has garnered substantial attention, to validate the importance of multi-dimensional inter-company relationships.

In 2021, *Evergrande Group*, a prominent player in the real estate industry, encountered a debt crisis that reverberated throughout the entire supply chain of the industry. Notably, its default on commercial debts had cascading effects: over 40 associated companies were affected, collectively representing a market value exceeding 370 billion yuan. Among these entities, an overwhelming majority of 44% experienced their first-ever financial losses.

As A-share listed companies disclosed their performance projections for 2021, several entities in upstream sectors of the real estate industry, including *Shanghai Electric* (currently involved in litigation), *Three Trees*, and *Boss Electric*, faced repercussions due to their exposure to *Evergrande Group*'s commercial debts. Notably, *Suzhou Gold Mantis Construction Decoration Co., Ltd.*, operating in the decoration industry, suffered significant losses for the first time as a result of its multifaceted connections with *Evergrande Group*. These losses are estimated to range between 4 to 5 billion yuan.

We compare the variation rates of net profit between the end of 2021 for five companies entwined with *Evergrande Group* through both capital and industry channels, and five companies solely linked with *Evergrande Group* through a single industry.

Table 3.3 illustrates that subsequent to the credit risk outbreak in *Evergrande Group*, companies with multiple associations exhibit negative year-on-year growth rates in net profit, whereas those with single associations predominantly display positive growth rates. This outcome validates that interconnectedness through multiple associations amplifies the impact of a risk event in one enterprise on other linked entities. It further underscores the importance of investigating the multi-dimensional relationships between enterprises.

Table 3.3 Net Profit Year-on-Year Growth Rates for Companies with Multi-dimensional and Single-dimensional Associations with *Evergrande Group*

Single association		Multiple association	
Company name abbreviation	Change rate of net profit year-on-year (%)	Company name abbreviation	Change rate of net profit year-on-year (%)
Shenzhen Magic Design & Decoration	11.46	*Shanghai Electric*	-10.28
Marssenger Kitchenware	0.38	*Wenke Green Technology*	-9.57
Zhengzhong design	0.34	*Jiayu Holding*	-2.81
Mingdiao Decoration	-0.34	*Gold Mantis Construction Decoration*	-2.87
Minkave Technology	-0.61	*Grandland Decoration*	-0.99

4 Conclusion and Outlook

This paper presents a novel methodology rooted in multilayer networks to discern intricate inter-firm associations and their structural attributes. Using the real estate sector as a case study, it delves into the nuanced web of inter-firm relationships and their evolutionary dynamics, while also conducting a comprehensive validation analysis through case studies.

The empirical findings unveil the intricate nature of inter-firm associations within the real estate domain, with industry-based networks exhibiting higher complexity compared to those based on capital connections. Importantly, enterprises entrenched within complex industry networks may not necessarily engage extensively in external shareholding or investment activities. The majority of real estate enterprises are intertwined with multiple industries, demonstrating a preference for diversified operations and adopting comprehensive management strategies. Furthermore, the significance of associations across different network layers evolves over time, with the importance of industry-centric connections surpassing that of capital-based linkages, particularly post-2015.

This study bears significant policy implications, especially in the context of contemporary macroeconomic priorities focusing on the prevention and mitigation of risks in critical sectors, with real estate risk mitigation remaining paramount. In this regard, the characterization of multiple inter-firm associations in the real estate sector as outlined in this paper offers valuable insights for understanding and mitigating the contagion of credit risks. These insights assist enterprises and regulatory bodies in accurately assessing the transmission pathways and exposure levels of risks, facilitating timely detection and response to risks, and preemptively signaling potential risk events to safeguard financial system stability.

The methodology proposed herein, provides a more nuanced depiction of complex inter-firm interactions, distinguishing itself from conventional association metrics and single-layer network analyses. It demonstrates applicability in analyzing multilayered complex networks that incorporate diverse association types, thereby improving the identification of intricate decision-making mechanisms among enterprises within a framework of multiple associations. Furthermore, it enables the examination of social-economic spillover effects and complex channels of financial risk transmission. As a result, this methodology holds promising prospects for both theoretical advancements and practical applications across domains such as behavioral finance, corporate finance, asset pricing, and risk management.

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