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Measuring Global Value Chain Risks Based on the Absorbing Markov Model with Rewards

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Abstract

Global Value Chains (GVCs) are central to our understanding of international trade and economic symbiosis. The dense interdependence that typifies these global networks, a consequence of accelerated globalization, makes them especially susceptible to various disruptions such as geopolitical tensions, energy scarcity shocks, and extreme climatic events. A disturbance in one part of the chain can rapidly lead to cascading effects, resulting in substantial economic impacts that transcend national boundaries. This study designs an Absorbing Markov Model with Rewards to trace the risks in GVCs. It employs the input-output table and integrates it with the absorbing Markov process to elucidate the flows within GVCs and a Markov Reward Process to quantify the transmission of risks within these chains, the study utilizes. This approach provides a method of quantifying how risks accompany the flow of goods and services, considering both the intensity and the propagation of risk factors. This general method makes it applicable to a wide range of aspects—whether analyzing the flow of value-added, energy, emissions, or other factors and how they pass through different key sectors (assuming the risk index to be identical), or analysing the multi-perspective risks, e.g., climate change risks, natural resources scarcity risks, geo-political risks, socio-economical risks, and many other aspects.

Keywords

Global Value Chains, Absorbing Markov Chains, Markov Rewards Model, Supply Chain Risks, Natural Disaster Risks

1. Introduction

In recent decades, Global Value Chains (GVCs) have become central to our understanding of international trade and economic symbiosis (Antràs, 2020). GVCs, functioning as complex networks for the production, assembly, and distribution of goods and services (Hummels et al., 2001), encapsulate more than just economic efficiency; they are pivotal in adapting to and dealing with diverse risks. The ever-increasing interdependence that typifies these global networks—a consequence of accelerated globalization—makes them especially susceptible to various disruptions. Geopolitical tensions, for instance, can trigger trade restrictions or resource nationalism, leading to supply chain disruptions and inflated costs (Chai et al., 2024). Energy/resource/food/labor supply shocks, arising from resource limitations, natural endowments, or even geopolitical issues, can significantly escalate production expenses and cause delays, reverberating throughout the chain and impacting end-user prices and availability (Anderson et al., 2007; Zhang et al., 2023). Moreover, natural disasters and extreme climatic events, such as floods, droughts, or hurricanes, have the potential to halt production, devastate infrastructure, or impede logistics (Li et al., 2022; Sun et al., 2024). In the realm of GVCs, a disturbance in one part of the chain can rapidly lead to cascading effects, resulting in substantial economic impacts that transcend national boundaries. Therefore, the precise measurement and effective management of these risks are crucial in ensuring economic resilience in a world that is deeply interconnected yet marked by uncertainty.

Assessing risks within GVCs is critical to build a resilient production and consumption system, however, it is also a challenging issue. The primary challenge lies in the inherent complexity and interconnectedness of the GVC networks. As GVCs span diverse geographical regions and encompass numerous processes, disruptions in each single node can trigger widespread and unpredictable effects upstream and downstream. This necessitates analytical methodologies to capture such interdependencies. Another notable challenge is that GVCs are exposed to a broad spectrum of risks, including economic, geopolitical, environmental, and social factors. An effective approach requires a methodology that is adaptable to the unique features and vulnerabilities of different types of risks. The development of such customizable framework represents a crucial area of focus for researchers and practitioners in the field. Furthermore, analysing GVCs risks is also challenging due to the lack of transparency of production, particularly at the micro-level or individual enterprises levels. In this context, the application of input-output analysis to understand economic interdependencies becomes particularly pertinent. By utilizing input-output models, researchers can uncover the structural patterns of GVCs, pinpointing segments most susceptible to risks.

Substantial progress has been made in applying input-output models to quantify risks within GVCs. These studies have adopted innovative approaches, each contributing uniquely to our understanding of GVC risk dynamics. Some researchers have focused on quantifying the impact of specific shocks, analyzing how changes in volume and production due to events like natural hazards can affect GVCs. This approach provides valuable insights into the immediate and subsequent effects of such disruptions. Another strand of research has employed techniques such as the hypothetical extraction method, centrality measures, and PageRank theory to identify the most crucial sectors within GVCs (Dietzenbacher & Lahr, 2013; Tokito et al., 2022). By pinpointing these key sectors, these studies offer a way to understand which parts of the chain are most influential and, therefore, potentially more vulnerable to risks. Additionally, there have been attempts to quantify risks based on the frequency with which GVCs intersect with sectors deemed risky. This perspective considers not just the impact of a risk event but also its likelihood, based on the GVC's exposure to high-risk sectors. However, based on Inomata & Hanaka (2021, 2024) pointed out, a comprehensive risk assessment in GVCs should consider the volume, the strength, and frequency dimensions. They likened this to the risk of virus infection and earthquake events. Here this study uses a similar analogy, where the chances that a factory will be exposed to natural disasters (e.g., hurricanes) more badly because (1) a large amount of the factory's production is exposed to hurricanes; (2) the hurricane is more severe; or (3) the hurricanes happen frequently. In light of these insights, this study aims to offer a more comprehensive framework for quantifying risks in GVCs. Our approach integrates the volume of value-added/energy/emissions passing through the GVCs, the frequency and pathways of these flows, and the inherent risk strength of the country-sectors involved. By combining these dimensions, this study seeks to provide a more nuanced understanding of the multifaceted nature of risks in GVCs.

This study uses an Absorbing Markov Model with Rewards to trace the risks in GVCs. It employs the input-output table and integrates it with the absorbing Markov process to elucidate the flows within GVCs. The Markov process offers a sequential representation of production from a probabilistic perspective, enabling us to illuminate how production is organized step by step and capture how different countries and sectors are involved and interconnected in various production chains. To quantify the transmission of risks within these chains, the study utilizes a Markov Reward Process. In this method, risks are incorporated into the GVCs by assigning a "risk index" to each country-sector (or to each country-sector- country-sector pair), which can be understood as "rewards" related to each step of move in the supply chain. It also considers a discount index to describe how risks penetrate or accumulate along these chains. This approach provides a nuanced method of quantifying how risks accompany the flow of goods and services, considering both the intensity and the propagation of risk factors. This study applies this model to analyse risks from various starting states (the origin of the chains) or absorbing states (the final products of the chains), between specific starting-absorbing states bilaterally, and through complete pathways from staring state to absorbing state, passing through intermediate risky nodes. This comprehensive application allows for a detailed risk assessment from multiple perspectives within the GVC framework. This general method makes it applicable to a wide range of aspects—whether analyzing the flow of value-added, energy, emissions, or other factors and how they pass through different key sectors (assuming the risk index to be identical), or analysing the multi-perspective risks, e.g., climate change risks, natural resources scarcity risks, geo-political risks, socio-economical risks, and etc. Thus, this study could offer both theoretical and practical insights into the realm of global supply chains.

2. Materials and methods

2.1 Input-output analysis

A multiregional input-output model describes how production is organized through transactions, which can be further extended to analyse embodied materials, resources, emissions, pollutions, as well as many other factors (Feng et al., 2013; Lenzen et al., 2022; Meng et al., 2023; Fu et al., 2023). Table 1 presents a MRIO table including m countries and n sectors. In a typical MRIO table, there are several key elements. The transaction matrix $\boldsymbol{Z} = [z_{ij}]$ $(i, j = 1,2, \dots, m \times n)$representing the input of a commodity from country-industry i to country-industry j, where $m \times n$ is the dimension of the intermediate transaction matrix. The gross outputs vector $\boldsymbol{X} = [x_i]$ representing the gross outputs of the country-sector pair. Then we could get the direct input coefficient matrix $\boldsymbol{A} = [a_{ij}]$ and the output coefficient matrix $\boldsymbol{B} = [b_{ij}]$ where $a_{ij} = z_{ij}/x_j$ denotes the input from i necessary for producing a unit of output of j and $b_{ij} = z_{ij}/x_i$ denotes the output from i to j for a unit of output of i. The final demand matrix $\boldsymbol{F} = \sum_m [f_{im}]$ and the value-added matrix $\boldsymbol{W}^T = [w_i]$ represent the final demand and value-added in the input-output system.

Two classic models can be applied to analyse the value, energy, emission, as well as other flows in the input-output system, i.e., the Leontief demand-pull model and the Ghosh input-driven model. While the Leontief demand-pull model is based on the Leontief inverse related to $\boldsymbol{A}$, the Ghosh input-driven model is based on the Ghosh inverse related to $\boldsymbol{B}$.

$$\boldsymbol{X} = \boldsymbol{L}\boldsymbol{Y} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{F}$$
$$\boldsymbol{X} = \boldsymbol{G}^T\boldsymbol{V} = (\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{W}$$

Table1. Input-output Table

		Input use				Final use				Total use
		1	2	⋯	M	1	2	⋯	M	
Output supplied	1	Z^{11}	Z^{12}	⋯	Z^{1m}	F^{11}	F^{12}	⋯	F^{1m}	X^1
	2	Z^{21}	Z^{22}	⋯	Z^{2m}	F^{21}	F^{22}	⋯	F^{2m}	X^2
	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋱	⋮	⋮
	M	Z^{m1}	Z^{m2}	⋯	Z^{mm}	F^{m1}	F^{m2}	⋯	F^{mm}	X^m

Value added	W^{1T}	W^{2T}		W^{mT}	
Gross output	X^{1T}	X^{2T}		X^{mT}	
Risk	R^1	R^2	$\cdots$	R^m	

2.2 Markov Process and Absorbing Markov Chains.

A Markov chain is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event (Kemeny & Snell, 1983). The states can be classified into two types of sets, transient (once left, never entered) or ergodic (once entered, never left). If a state is the only element in an ergodic set, then it is called an absorbing state, which has transition probability to itself of 1. A chain with all its non-transient states absorbing, is an absorbing chain. In other words, the Markov chain is an absorbing Markov chain if there exists at least one absorbing state. Given an absorbing Markov chain with u transient states and v absorbing states. Let $\boldsymbol{\mathcal{M}} = \{\mathcal{X}_0, \mathcal{X}_1, \mathcal{X}_2, \dots\}$ be an absorbing Markov chain with state space $S = U \cup V$. Let $\boldsymbol{T} = [t_{ij}]$ denotes the transition probability of transition from state i to state j, then we have

$$t_{ij} = \mathbb{P}(\mathcal{X}_{t+1} = j | \mathcal{X}_t = i) \quad for\ i, j \in S, \ \ t = 0,1,2, \dots$$

$$\mathbb{P}(\mathcal{X}_{t+1} = j | \mathcal{X}_t = i, \mathcal{X}_t = s_{t-1}, \mathcal{X}_{t-2} = s_{t-2}, \dots) = \mathbb{P}(\mathcal{X}_{t+1} = j | \mathcal{X}_t = i)$$

Arranging the absorbing Markov chain in a canonical form as follows,

$$\boldsymbol{T} = \begin{pmatrix} \boldsymbol{Q} & \boldsymbol{P} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix}$$

where Q is a $u \times u$ block containing the transition probabilities between transient states, P is a $u \times v$ block containing the transition probabilities from transient to absorbing states, I is the identity matrix of order v, and 0 is a block with null elements. The transition matrix describes how initial inputs are transmitted and absorbed by final demand as follows,

$$\boldsymbol{T}^k = \begin{pmatrix} \boldsymbol{Q^k} & \boldsymbol{R} + \boldsymbol{QR} + \cdots + \boldsymbol{Q^{k-1}R} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix}$$

$$\lim_{k \to \infty} \boldsymbol{T}^k = \begin{pmatrix} \boldsymbol{0} & (\boldsymbol{I} - \boldsymbol{Q})^{-1}\boldsymbol{R} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix}$$

Thus, we get the fundamental matrix $\boldsymbol{N}$ of dimension $u \times u$,

$$\boldsymbol{N} = (\boldsymbol{I} - \boldsymbol{Q})^{-1}$$

where n_{ij} is the expected number of times the chain is in state j given that it starts from state i. Thus, the expected number of intermediate steps before being absorbed is the row sums of $\boldsymbol{N}$, which can be expressed as

$$n_i = \sum_j n_{ij}$$

$\boldsymbol{D}$ is an $u \times v$ matrix whose element d_{ij} is the probability that an absorbing chain will be absorbed in the absorbing state j if it starts in the transient state i.

$$\boldsymbol{D} = (\boldsymbol{I} - \boldsymbol{Q})^{-1}\boldsymbol{R} = \boldsymbol{NR} = \begin{pmatrix} n_{11} & \cdots & n_{1t} \\ \vdots & \ddots & \vdots \\ n_{t1} & \cdots & n_{tt} \end{pmatrix} \begin{pmatrix} r_{11} & \cdots & r_{1r} \\ \vdots & \ddots & \vdots \\ r_{t1} & \cdots & r_{tr} \end{pmatrix}$$

We can also understand the element d_{ij} as the sum of probability that something starting from state i to be absorbed by state j through all possible last state k. As $\frac{n_{ik}}{n_i}$ can be understood as the possibility of being in state $k \in \{1,2,\ldots,u\}$ of anything starts from state i before final absorption, while r_{kj} can be understood as the probability of something in state k that goes to absorbing state j. $\boldsymbol{D}$ is a row-random matrix.

As the input-output model also describes the production process with transitions between intermediate production states (transient states) and finally consumed as final product (absorbing states). The number of transient states equal the number of country-sectors $u = m \times n$ and the number of absorbing states also equal the number of country-sectors of final products $v = m \times n$. The input-output model can also be viewed from the probability-based perspective, such as random walk with probabilities on the world GVCs network (Piccardi et al., 2018).Thus, we could also use the absorbing Markov chain to describe the supply chains of the input-output model, where the transition matrix can be defined as the transition probabilities between intermediate input/outputs and final products (Wirkierman et al., 2022).

$$\boldsymbol{T_B} = \begin{pmatrix} \boldsymbol{B} & \boldsymbol{f} \\ \boldsymbol{0} & \boldsymbol{I_n} \end{pmatrix}, \boldsymbol{N_B} = (\boldsymbol{I} - \boldsymbol{B})^{-1}, \boldsymbol{D_B} = (\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{f}$$

$$\boldsymbol{T_A} = \begin{pmatrix} \boldsymbol{A}^T & \boldsymbol{w} \\ \boldsymbol{0} & \boldsymbol{I_n} \end{pmatrix}, \boldsymbol{N_A} = (\boldsymbol{I} - \boldsymbol{A}^T)^{-1}, \boldsymbol{D_A} = (\boldsymbol{I} - \boldsymbol{A}^T)^{-1}\boldsymbol{w}$$

where $\boldsymbol{T_B}$, $\boldsymbol{N_B}$, and $\boldsymbol{D_B}$ describe where the value-added goes to, e.g., $d_{b_{ij}}$ represents the share of value-added of country-sector i that is absorbed by country-sector j; while $\boldsymbol{T_A}$, $\boldsymbol{N_A}$, and $\boldsymbol{D_A}$ describe where the value-added originates, e.g., $d_{a_{ij}}$ represents the share of country-sector i's final product that originates from country-sector j's value-added. Table 2 shows how a classic input-output table can be also expressed as an Absorbing Markov Chain (Duchin & Levine, 2010; Kostoska et al., 2020; Moosavi & Isacchini, 2017). A few previous studies have used the probability-based Markov Chain model to analyse input-output problems. For instance, Duchin & Levine (2010) used a Markovian framework to analyse the embodied resources flows. Kostoska et al. (2020) applied the Markovian method to analyse the structure and lengths of value chains.

Table 2. An Absorbing Markov Chain for the Input-output Table

	Transient States				Absorbing States			
	1	2	⋯	u	1	2	⋯	v

Transient States	1	b_{11}	b_{12}	$\cdots$	b_{1u}	y_{11}	y_{12}	$\cdots$	y_{1v}
	2	b_{21}	b_{22}	$\cdots$	b_{2u}	y_{21}	y_{22}	$\cdots$	y_{2v}
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	u	b_{u1}	b_{u2}	$\cdots$	b_{uu}	y_{u1}	y_{u2}	$\cdots$	y_{vv}
Absorbing States	1	0	0	$\cdots$	0	1	0	$\cdots$	0
	2	0	0	$\cdots$	0	0	1	$\cdots$	0
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	v	0	0	$\cdots$	0	0	0	$\cdots$	1
Risk		r^1	r^2	$\cdots$	r^m				

2.3 Markov Reward Process

A Markov Reward Process is an extension of the basic Markov process, where each state in the process is associated with a reward or risk (Cardoso et al., 2019). In a Markov Reward Process, the future state and the immediate reward depend only on the current state, adhering to the Markov property. In a Markov Reward Process, each state transition in the Markov chain is accompanied by a numerical reward (or risk), adding an additional layer of value to the state changes. This reward (risk) structure allows for the quantification of the “value” of being in a particular state, considering not just the current reward but also the expected future rewards. Markov Reward Processes are particularly useful in modeling scenarios where decision outcomes or state-changing have both immediate and long-term consequences, as they provide a framework for understanding how immediate state transitions impact future states and their associated rewards. The cumulative reward over time, often analyzed through concepts like expected return, offers a comprehensive view of the process’s evolution, making Markov Reward Process a powerful tool in areas where outcomes are probabilistic and interconnected.

One famous Markov Reward Process is the frozen-lake problem in Figure 1, where a tiny soldier moves on the frozen lake from cell to cell with various probabilities and some cells are related to a risk of falling into the cold winter lake. The input-output model can be likened to a more complex version of the frozen lake problem. Imagine each cell on the frozen lake as a specific country-sector in the economy. Just as the soldier moves from cell to cell, economic resources move between different production states. In the input-output model, these probabilities are analogous to the technical coefficients or the transition probability matrix like $\boldsymbol{T_B}$ or $\boldsymbol{T_A}$. The transition from one country-sector to the other mirrors how the little soldier's movement from one cell (sector) to another depends on the conditions of the current cell. Just as some cells on the frozen lake carry a risk of the soldier falling through, certain country-sectors in an economy can be associated with various levels of risk. These risks could be economic (like volatility in demand or supply), environmental or climatical (such as resource depletion, extreme climate events, and natural disasters), or even geopolitical (like trade wars or even wars). The assessment and management of these risks are crucial for sustaining economic stability and growth, similar to how the soldier's strategy must account for the risky cells to safely navigate the lake.

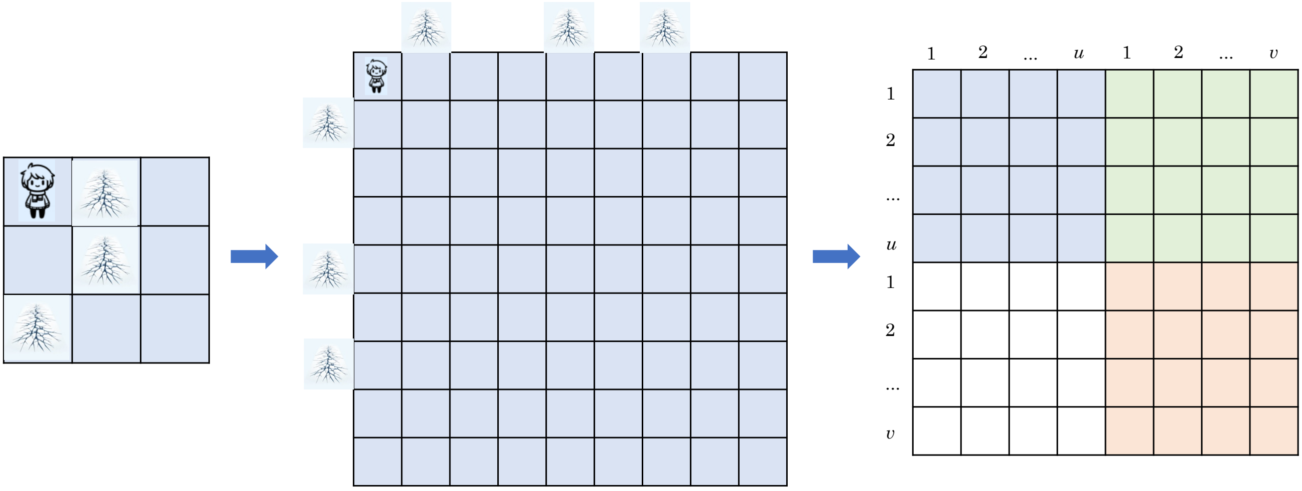


Figure 1. From the Frozen-lake problem to Markov Reward Process for the Input-output Model. The little human is moving on a frozen lake. Some parts of the lake is frozen and safe, while some parts of the lake are covered with broken ice, indicating a hidden risk of falling into the lake.

2.4 Input-output model and Absorbing Markov Chain with Rewards

In this section, we define the input-output model using an Absorbing Markov Chain with Rewards (AMCR). In this AMCR, the state space is defined as $S = U \cup V$, where $U = \{1,2,\dots,u\}$ and $V = \{1,2,\dots,v\}$ are non-empty disjoint sets comprised of the transient and absorbing states of S, respectively. Each transient state is a country-sector in the production process and each absorbing state is a country-sector for final product.

As the input-output model is symmetrical when analysed using the Leontief and the Ghosh model, here we use the output coefficient matrix $\boldsymbol{B}$ as an example. The transition probability matrix is as follows,

$$\boldsymbol{T} = \begin{pmatrix} \boldsymbol{B} & \boldsymbol{F} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix} = \begin{pmatrix} \boldsymbol{Q} & \boldsymbol{P} \\ \boldsymbol{0} & \boldsymbol{I} \end{pmatrix}$$

To quantify the risks related to the supply chains, we assume a risk function $R(t)$ for the absorbing Markov chain. To capture the impact of upstream and downstream risk, we also consider a discount index $\gamma \in [0,1]$. When $\gamma = 0$, it means the risks will not accumulate along the supply chains, that is, the supply chain is only impact by the direct risk of the current state. When $\gamma = 1$, the risks fully accumulate along the supply chains, that is, the upstream/downstream risks will be taken into fully account. Thus, the total risk of the process from time t can be expressed as follows,

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

Then, for each state $s \in S$, the related risk $V(s)$ is the total risk for the process starting from state s, so that

$$V(s) = E[G_t | S_t = s]$$

Based on this absorbing Markov chain with rewards model, we can quantify the risk of the global supply chains.

2.4.1 Risk related to each starting state before absorption (1D-risk)

First, we will try to quantify the risk related to each starting state before absorption. Define u_j^k as a function of the process,

$$u_j^k = \begin{cases} 1, & \textit{the process is in state } s_j \textit{ after } k \textit{ steps} \\ 0, & \textit{othersise} \end{cases}$$

Then the total risk of the process from being in state s_j can be expressed as the sum of risk related to state s_j

$$n_j = \sum_{k=0}^{\infty} \gamma^k u_j^k r_j$$

Denote the mean value of n_j as $M[n_j]$. Then, the expected risk starting from state s_i by visiting state s_j can be expressed as $\boldsymbol{N} = \{\boldsymbol{M}_i[n_j]\}$,

$$\begin{aligned}
\boldsymbol{N} &= \{\boldsymbol{M}_i[n_j]\} \\
&= \left\{\boldsymbol{M}_i\left[\sum_{k=0}^{\infty} \gamma^k u_j^k r_j\right]\right\} \\
&= \left\{\sum_{k=0}^{\infty}\left(\left(1 - p_{ij}^{(k)}\right) \cdot 0 + p_{ij}^{(k)} \gamma^k r_j\right)\right\} \\
&= \sum_{k=0}^{\infty}\left\{p_{ij}^{(k)} \gamma^k r_j\right\} \\
&= \sum_{k=0}^{\infty} \gamma^k \boldsymbol{Q}^k \boldsymbol{R} \\
&= (\boldsymbol{I} - \gamma \boldsymbol{Q})^{-1} \boldsymbol{R}
\end{aligned}$$

Denote $\boldsymbol{\xi}$ as a column vector with elements 1, the total expected risk staring from each state can be obtained as row sums of $\boldsymbol{N}$ as follows

$$\boldsymbol{\tau} = \boldsymbol{N\xi}$$

To make the question simpler, we first assume that the risk of each transition is only related to the state it moves to, thus the risk matrix can be expressed as a diagonal matrix $\boldsymbol{R} = \{\widehat{r_j}\}$.

(1) When $\gamma = 1$ and $\boldsymbol{R} = \boldsymbol{I}$ (fully accumulation of upstream/downstream risk; risks are homogeneous across country-sectors and normalized), $\boldsymbol{N}$ reduces to $\boldsymbol{N}_1 = (\boldsymbol{I} - \boldsymbol{Q})^{-1}$. We

can see that n_i is the total number of steps needed to reach an absorbing state for a chain staring from state s_i and n_{ij} is the total number of steps in state s_j for a chain staring from state s_i before absorption. These are consistent with previous conclusions of a normal absorbing Markov chain.

(2) When $\gamma = 0$, $\boldsymbol{N}$ reduces to $\boldsymbol{R}$, which is the risk only related to the starting state.

(3) When only a subset of states S_r are considered as risky, we can denote the new risk matrix as $\boldsymbol{R}_r$, where $r_{jj} = 1\ iff\ s_j \in S_r\ and\ 0\ otherwise$. Thus, $\boldsymbol{N}$ can be written as $(\boldsymbol{I}-\boldsymbol{Q})^{-1}\boldsymbol{R}_r$, which keeps only the risky subsets. From this expression, we could analyse the risk related to passing-through certain countries/regions of interest.

2.4.2 Probability of captured by each absorbing state (2D-distribution)

Then, we try to answer the question about which absorbing state is likely to capture the process. If $\boldsymbol{D} = \{d_{ij}\}$ is the probability that the process starting in transient state $s_i \in U$ ends up in absorbing state $s_j \in V$. Starting from s_i, the process may be captured in s_j in one or more steps. The probability of capture on a single step is p_{ij}. If this does not happen, the process may move either to another absorbing state (in which case it is impossible to reach s_j), or to a transient state s_k. In the latter case there is probability d_{kj} of being captured in the state s_j that we concern. Thus,

$$d_{ij} = p_{ij} + \sum\nolimits_{s_k \in U} p_{ik} d_{kj}$$

$$\boldsymbol{D} = \boldsymbol{P} + \boldsymbol{Q}\boldsymbol{D}$$

$$\boldsymbol{D} = (\boldsymbol{I}-\boldsymbol{Q})^{-1}\boldsymbol{P} = \boldsymbol{N}_1\boldsymbol{P}$$

An alternative method to calculate $\boldsymbol{D}$ is to consider every time that the process is in transient state s_k, it has probability p_{kj} of going to s_j. Hence it is possible to show that

$$d_{ij} = \sum_{s_k \in U} \boldsymbol{M}_i[n_{1_k}] p_{kj}$$

$$\boldsymbol{D} = \boldsymbol{N}_1\boldsymbol{P}$$

It is worth noting that the row sum of $\boldsymbol{D}$ is always 1, as the total possibility of being absorbed is 1. If we are interested in one specific absorbing state $s_a \in V$, we could obtain the probabilities of absorption in the given absorbing state s_a for any transient state as initial state as follows:

$$\boldsymbol{d}_a = \boldsymbol{N}_1\boldsymbol{p}_a$$

where $\boldsymbol{p}_a$ is the column vector in $\boldsymbol{P}$ concerning absorbing state s_a.

2.4.3 Risks between each starting state and absorbing state (2D-risk)

Then, if we are interested in the risk related to both the specific starting state and specific absorbing state, we could analyse in more detail. Still, the absorbing state that we concern is state $s_a \in V$. All processes that start from any state $s_i \in U$ and ends up in state s_a surely forms an absorbing Markov chain with a single absorbing state. The transient states are the same as those in the original absorbing Markov chain with multiple absorbing states, but the transition probabilities are different as other absorbing states are not included in this chain (so that the chain will not "fall" in other "holes").

Denote f_n as the state that the process is at after the nth step. For instance, $f_1 = s_k$ if the process is at state s_k after moving one step. Denote p as the event that the process is absorbed in state s_a in the original process. Then, the new one-step transition probability from state s_i to s_j in this Markov chain with single absorbing state can be estimated according to the conditional probabilities as follows:

$$\begin{aligned} p'_{ij} &= \mathbb{P}_i[f_1 = s_j|p] \\ &= \frac{\mathbb{P}_i[f_1 = s_j \wedge p]}{\mathbb{P}_i[p]} \\ &= \frac{\mathbb{P}_i[p|f_1 = s_j]\,\mathbb{P}_i[f_1 = s_j]}{\mathbb{P}\mathrm{r}_i[p]} \\ &= \frac{d_{ja}p_{ij}}{d_{ia}} \end{aligned}$$

The new transition matrix can be expressed in matrix form as follows,

$$\boldsymbol{Q}' = \widehat{\boldsymbol{d}_a}^{-1}\boldsymbol{Q}\widehat{\boldsymbol{d}_a}$$

$$\boldsymbol{P}' = \frac{\boldsymbol{P}_a}{\boldsymbol{d}_a}$$

Thus, the new fundamental matrix to calculate the number of steps before absorption is

$$\begin{aligned} \boldsymbol{N}_{1a} &= \boldsymbol{I} + \boldsymbol{Q}' + \boldsymbol{Q}'^2 + \cdots \\ &= \boldsymbol{I} + \widehat{\boldsymbol{d}_a}^{-1}\boldsymbol{Q}\widehat{\boldsymbol{d}_a} + \widehat{\boldsymbol{d}_a}^{-1}\boldsymbol{Q}^2\widehat{\boldsymbol{d}_a} + \cdots \\ &= \widehat{\boldsymbol{d}_a}^{-1}(\boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^2 + \cdots)\widehat{\boldsymbol{d}_a} \\ &= \widehat{\boldsymbol{d}_a}^{-1}(\boldsymbol{I} - \boldsymbol{Q})^{-1}\widehat{\boldsymbol{d}_a} \end{aligned}$$

where $\boldsymbol{d}_a$ is the $a-th$ column of matrix $\boldsymbol{D}$ which is related to absorbing state s_a, and $\widehat{\boldsymbol{d}_a}$ be a diagonal matrix with diagonal entries d_{ja} and $\widehat{\boldsymbol{d}_a}^{-1}$ be its inverse. $\boldsymbol{N}_{1a} = \{n_{a_{ij}}\}$ represents the average number of steps for value-added starting from state s_i spent in state s_j to be absorbed as final products in state s_a.

After quantifying the number of steps between each staring-absorbing state pair, we can further consider the related risks. Remembering that the risk from being in state s_j can be expressed as n_j, the expected risk starting from state s_i and finally absorbed by s_a by visiting state s_j can be expressed as follows,

$$
\begin{aligned}
\boldsymbol{N}_a &= \{\boldsymbol{M}_{ia}[n_j]\} \\
&= \left\{\boldsymbol{M}_{ia}\left[\sum_{k=0}^{\infty} \gamma^k u_j^k r_j\right]\right\} \\
&= \left\{\sum_{k=0}^{\infty} \left(\left(1 - {p'}_{ij}^{(k)}\right) \cdot 0 + \gamma^k {p'}_{ij}^{(k)} r_j\right)\right\} \\
&= \sum_{k=0}^{\infty} \left\{\gamma^k {p'}_{ij}^{(k)} r_j\right\} \\
&= \sum_{k=0}^{\infty} \gamma^k \boldsymbol{Q}'^k \boldsymbol{R} \\
&= \widehat{\boldsymbol{d}_a}^{-1}(\boldsymbol{I} - \gamma\boldsymbol{Q})^{-1}\widehat{\boldsymbol{d}_a}\boldsymbol{R}
\end{aligned}
$$

(1) When there is only one absorbing state in the original Markov chain, i.e., $\boldsymbol{D} = \boldsymbol{\xi}$, the original Markov chain and the single absorbing state Markov chain gives the same results as $\boldsymbol{N}_a = \hat{\boldsymbol{\xi}}^{-1}(\boldsymbol{I} - \gamma\boldsymbol{Q})^{-1}\hat{\boldsymbol{\xi}}\boldsymbol{R} = (I - \gamma\boldsymbol{Q})^{-1}\boldsymbol{R}$.

(2) When $\gamma = 1$ and $\boldsymbol{R} = \boldsymbol{I}$ (fully accumulation of upstream/downstream risk; risks are homogeneous across country-sectors and normalized), $\boldsymbol{N}_a$ reduces to $\widehat{\boldsymbol{d}_a}^{-1}(\boldsymbol{I} - \boldsymbol{Q})^{-1}\widehat{\boldsymbol{d}_a}$. This gives the total expected number of steps between starting state s_i and absorbing state s_a.

(3) When $\gamma = 0$, $\boldsymbol{N}_a$ reduces to $\boldsymbol{R}$, which is the risk of the starting state. We can see if there is no accumulation/penetration effect, the risk is only related to the staring state s_i regardless of the absorbing state s_a.

(4) When only a subset of states S_r are considered as risky, we can denote the new risk matrix as $\boldsymbol{R}_r$, where $r_{jj} = 1\ iff\ s_j \in S_r\ and\ 0\ otherwise$. Then, $\boldsymbol{N}_a$ can be written as $\widehat{\boldsymbol{d}_a}^{-1}(\boldsymbol{I} - \gamma\boldsymbol{Q})^{-1}\widehat{\boldsymbol{d}_a}\boldsymbol{R}_r$, which keeps only the risky subsets. From this expression, we could analyse the risk related to passing-through certain countries/regions of interest.

(5) When we are interested in not only the strength of the risk (the average risk for something starting from each $s_i \in U$ absorbed by each $s_j \in V$) but also the possibility of the risk (the probability of something starting from each $s_i \in U$ to be absorbed by each $s_j \in V$), we further combine the results across different absorbing states. The total risks for value-added starting from state s_i to be absorbed in state s_a can be expressed as

$$\boldsymbol{\varphi}_a = \boldsymbol{N}_a\boldsymbol{\xi}$$

Combing all $\boldsymbol{\varphi}_j$ for absorbing states from 1 to v, we have the matrix $\boldsymbol{\varphi} = \{\varphi_{ij}\}$ for the risk starting from each state s_i to each absorbing state s_j.

$$\boldsymbol{\varphi} = (\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_v)$$

Then we consider the risks between two states, combining the risk strength between starting-absorbing pair matters and the probability distribution of value flow (or resources/energy/… under the extended input-output model) between them. In other words, the elements in $\boldsymbol{\varphi}$ cannot be added directly as these "events" have different possibilities to happen. That is, $\boldsymbol{\varphi} = \{\varphi_{ij}\}$ gives the risk for each unit of value starting from state s_i and absorbed in state s_j; however, we still need to link this to the probability of value starting from state s_i and absorbed in state s_j. Thus, the probability-weighted risk should be obtained by multiplying the expected number of steps and the related probability between the state pair as follows,

$$\boldsymbol{\phi} = (\boldsymbol{\varphi} \circ \boldsymbol{D})$$

where $\boldsymbol{\phi} = \{\phi_{ij}\}$ is the Hadamard product of the risk matrix and the probability matrix, which gives the risk distribution by absorbing country-sector s_j for each unit of initial input for each country-sector s_i. To assess the risk related to each starting country-sector, now we can use the row sum of $\boldsymbol{\phi}$,

$$\boldsymbol{\psi} = \boldsymbol{\phi}\boldsymbol{\xi}$$

$\boldsymbol{\psi} = \{\psi_i\}$ is a vector that represents the total risks related to each unit of input originating from s_i.

2.4.4 Risks by passing-through specific routes

In the previous section, we quantified the risk related to a specific starting state, a specific starting-absorbing state pair, and by passing-through any country-sector(s) of interest. In previous analysis, the risk matrix $\boldsymbol{R}$ was a diagonal matrix: a homogeneous and normalized risk index when quantifying the total number of steps ($\boldsymbol{R} = \hat{\boldsymbol{\xi}}$) and a heterogenous on country level or country-sector level when quantifying the total related risks of passing-through certain production stages ($\boldsymbol{R} = \{\hat{r}_j\}$).

Here, we try to further expand the results to a more generalized version by taking into consideration the risks related to passing-through specific routes. That is, instead of solely dependent on the country-sector it passes ($s_k\ in\ step\ k$), the risk in this section the specific routes it passes ($s_{k-(k+1)}\ in\ step\ k+1$). For instance, in previous analysis where risks are solely dependent on discrete states, a transition from "🍌banana(risk=1)—🍎apple(risk=2)—🍊orange(risk=3)" will results in total risk of 6 when risks are completely accumulative and penetrative. Other transitions like "🍌—🍊—🍎", "🍊—🍎—🍌", "🍊—🍌—🍎", "🍎—🍊—🍌", and "🍎—🍌—🍊" all results in the same level of total risks. However, this is not necessarily true. The routes "🍌—🍎—🍊" and "🍌—🍊—🍎" could have different risks if the relationship between these states are of various risk levels.

To capture the route effect, $\boldsymbol{R}$ is no longer a diagonal matrix, but a matrix with element r_{ij} for transition between each s_{j-l} state pair. The event of passing-through route s_{j-l} can be understood as two consecutive events: (1) passing-through state s_j; (2) transition from s_j to s_l at the next step. Which means that the probability of passing-through route s_{j-l} at step $k+1$ is the probability of passing-through s_j at step k times the probability of transition from s_j to state s_l. Thus, the total risk originating from state s_i and finally absorbed by s_a by passing-through route "state s_j—state s_l" can be expressed as follows,

$$\begin{aligned}
\boldsymbol{N}_a^r &= \{\boldsymbol{M}_{ia}[n_{jl}]\} \\
&= \left\{\boldsymbol{M}_{ia}\left[\sum\nolimits_{k=0}^{\infty}\sum\nolimits_{l\in T}\gamma^{k+1}u_j^k r_{jl} + \sum\nolimits_{l\in T} r_{jj}\right]\right\} \\
&= \left\{\sum\nolimits_{k=0}^{\infty}\sum\nolimits_{l\in T}\left(\left(1-p'^{(k)}_{ij}\right)\cdot 0 + \gamma^{k+1}p'^{(k)}_{ij}p'_{jl}r_{jl}\right) + \sum\nolimits_{l\in T} r_{jj}\right\} \\
&= \sum\nolimits_{k=0}^{\infty}\sum\nolimits_{l\in T}\left\{\gamma^{k+1}p'^{(k)}_{ij}p'_{jl}r_{jl}\right\} + \sum\nolimits_{l\in T}\{r_{jj}\} \\
&= \sum\nolimits_{k=0}^{\infty}\gamma^{k+1}\boldsymbol{Q}'^{k}(\boldsymbol{Q}'\circ\boldsymbol{R}) + (\boldsymbol{I}\circ\boldsymbol{R}) \\
&= \gamma\widehat{\boldsymbol{d}_a}^{-1}(\boldsymbol{I}-\gamma\boldsymbol{Q})^{-1}\widehat{\boldsymbol{d}_a}\left((\widehat{\boldsymbol{d}_a}^{-1}\boldsymbol{Q}\widehat{\boldsymbol{d}_a})\circ\boldsymbol{R}\right) + (\boldsymbol{I}\circ\boldsymbol{R})
\end{aligned}$$

(1) When $\boldsymbol{R}$ have the same values in each column, i.e., the risk is only dependent on the next state the Markov moves to, this expression should give the same results with previous analysis. Denote $\boldsymbol{R}_{dg}$ as the diagonal matrix build from any row of $\boldsymbol{R}$. Do $\boldsymbol{N}_a$ and $\boldsymbol{N}_a^r$ present the same results? The answer is yes.

$$\boldsymbol{N}_a^r = \widehat{\boldsymbol{d}_a}^{-1}[\gamma(\boldsymbol{I}-\gamma\boldsymbol{Q})^{-1}\boldsymbol{Q}+\boldsymbol{I}]\widehat{\boldsymbol{d}_a}\boldsymbol{R}_{dg}$$

$$\boldsymbol{N}_a = \widehat{\boldsymbol{d}_a}^{-1}(\boldsymbol{I}-\gamma\boldsymbol{Q})^{-1}\widehat{\boldsymbol{d}_a}\boldsymbol{R}_{dg}$$

$$\boldsymbol{I}+\gamma(\boldsymbol{I}-\gamma\boldsymbol{Q})^{-1}\boldsymbol{Q} = \boldsymbol{I}+\gamma(\boldsymbol{I}+\gamma\boldsymbol{Q}+\gamma^2\boldsymbol{Q}^2+\cdots)\boldsymbol{Q} = \boldsymbol{I}+\gamma\boldsymbol{Q}+\gamma^2\boldsymbol{Q}^2+\cdots = (\boldsymbol{I}-\gamma\boldsymbol{Q})^{-1}$$

$$\boldsymbol{N}_a^r = \boldsymbol{N}_a$$

(2) When there is only one absorbing state in the original Markov chain, i.e., $\boldsymbol{D}=\boldsymbol{\xi}$, $\boldsymbol{N}_a^r$ can be expressed as

$$\gamma(\boldsymbol{I}-\gamma\boldsymbol{Q})^{-1}(\boldsymbol{Q}\circ\boldsymbol{R}) + (\boldsymbol{I}\circ\boldsymbol{R})$$

When the risk is only dependent on the arriving state of the Markov process ($\boldsymbol{R}$ have the same values in each column), the above expression could be further reduced to $\gamma(\boldsymbol{I}-\gamma\boldsymbol{Q})^{-1}\boldsymbol{Q}\boldsymbol{R}_{dg}+\boldsymbol{R}_{dg} = (\boldsymbol{I}-\gamma\boldsymbol{Q})^{-1}\boldsymbol{R}_{dg}$.

(3) When $\gamma = 1$ and $\boldsymbol{R}_{dg} = \boldsymbol{I}$ (fully accumulation of upstream/downstream risk; risks are homogeneous across country-sectors and normalized), $\boldsymbol{N}_a^r$ reduces to $\widehat{\boldsymbol{d}_a}^{-1}(\boldsymbol{I}-\boldsymbol{Q})^{-1}\widehat{\boldsymbol{d}_a}$. This still gives the total expected number of steps between starting state s_i and absorbing state s_a.

(4) When $\gamma = 0$, $\boldsymbol{N}_a^r$ reduces to $\boldsymbol{I} \circ \boldsymbol{R}$. This is the "self-self" transition risk. We can see if there is no accumulation/penetration effect, the risk is only related to the staring state s_i regardless of the absorbing state s_a.

(5) When only a subset of routes S_{pq} $p, q \in U$ as are considered as risky, we can denote the new risk matrix as $\boldsymbol{R}_{pq}$, where $r_{ij} = 1\ iff\ s_{ij} \in S_{pq}\ and\ 0\ otherwise$. $\boldsymbol{N}_a^r$ can be expressed as $\gamma \widehat{\boldsymbol{d}_a}^{-1}(\boldsymbol{I} - \gamma \boldsymbol{Q})^{-1}\widehat{\boldsymbol{d}_a}\left((\widehat{\boldsymbol{d}_a}^{-1}\boldsymbol{Q}\widehat{\boldsymbol{d}_a}) \circ \boldsymbol{R}_{pq}\right) + \left(\boldsymbol{I} \circ \boldsymbol{R}_{pq}\right)$, which keeps only the risky routes.

2.5 Data

To analyse how risks are transmitted in the global value chains, we apply the method to a specific kind of risk—the natural risk caused by disasters. Specifically, this study uses the World Risk Index (WRI) $\boldsymbol{R}$ developed by the Institute for International Law of Peace and Armed Conflict (Welle & Birkmann, 2015). The WRI takes into account both external and internal factors. A risk level is estimated for each country in each year from 2000 to 2023 based on both exposition to natural disasters (including Earthquakes, Tsunamis, Coastal Floodings, Riverine Floodings, Cyclones, Droughts, Sea Level Rise) and vulnerability (integrate assessment according to multi-dimensions and nearly 100 indicators on Susceptibility, Lack of Coping Capacities, and Lack of Adaptive Capacities). The WRI data can be accessed at https://data.humdata.org/dataset/worldriskindex?. The global multiregional input-output table is from OECD-ICIO (version 2021), with 67 countries and 45 sectors (further analysis will be conducted on version 2021 with 76 countries and 45 sectors). The OECD-ICIO table can be access at https://www.oecd.org/sti/ind/inter-country-input-output-tables.htm (OECD, 2021). To provide a picture of how this study provides a general method in input-output analysis, we also use an identity matrix $\boldsymbol{I}$ instead of $\boldsymbol{R}$ to see how this method can be applied to analyse length, upstreamness/downstreamness, as well as passing-through frequency.

3. Results

3.1 Risk for each country-sector ($\boldsymbol{R}_i$)

The GVCs not only link production stages but also transmit related risks. To see how downstream (upstream) risks accumulate along the GVCs, Figure 2 shows the expected total risks for each sector in respect to different risk deflators ranging from 0 to 1. We present the results of six selected countries, including the United States, Germany, Japan, China, India, and Russia. It is worth noticing that when risk deflator γ equals 0, the expected total risk would be exactly the direct risk of the certain country-sector; however, when the risk deflator increases, the total expected risk increases as well since indirect risks from value chains can penetrate and accumulate; and when the risk deflator equals 1, the risks fully accumulate along the supply chains.

Building on the foundational understanding of how GVCs not only link production stages but also serve as conduits for associated risks, we delve into the dynamics of risk transmission. Figure 2 shows the risk level of each country-sector as starting state for risk deflator γ of 0, 0.2 ,0.4, 0.6, 0.8, and 1. The risk level increases with the risk deflator γ. Figure 3 further shows the relationship between the expected total risks for each sector and varying levels of risk deflators, denoted by γ, ranging from 0, indicating no risk transmission, to 1, representing full risk permeation. The analysis presented focuses on six pivotal countries: the United States, Germany, Japan, China, India, and Russia. In our analysis, a pivotal observation is the non-linear escalation in total expected risk correlated with the increase of the risk deflator. As risk deflator increases, signifying greater risk transmission along the GVCs, we observe an intensified risk accumulation, particularly in sectors with extensive and fragmented production stages. It prompts a critical inquiry into the feasibility of achieving an optimized, yet secure, production framework that not only maximizes efficiency but also contains mechanisms to curtail risk penetration. Certain sectors demonstrate heightened sensitivity to the indirect risks propagated via GVCs. For instance, sectors such as D05T06, D07T08, and D09, which frequently engage with risk-laden production stages or regions, exhibit higher sensitivity. Such heightened risk could be induced by longer production chains and traversing through high-risk nodes more frequently. These findings underscore the importance of identifying and fortifying the more vulnerable segments within GVCs to enhance the overall resilience of the production network. Also, the relative risk levels exhibit potential reversals as the risk deflator intensifies. With no risk accumulation effect, we might observe a risk hierarchy where Country-Sector A incurs lower risks than Country-Sector B. However, as the risk deflator increases, this risk hierarchy may invert, with Country-Sector A surpassing B in total expected risk. This phenomenon is exemplified in the case of China and India across sectors D01T02, D03, D28, D29, D10T12, and D13T15, where China's risk level overtakes that of India as the risk deflator escalates. This reversal highlights the variable impact of integrated risks within GVCs and the significance of understanding how indirect risks can alter the comparative risk profiles of country-sectors.

Figure 2. Risk level of each country-sector as starting state ($\gamma = 0, 0.2, 0.4, 0.6, 0.8, 1$).

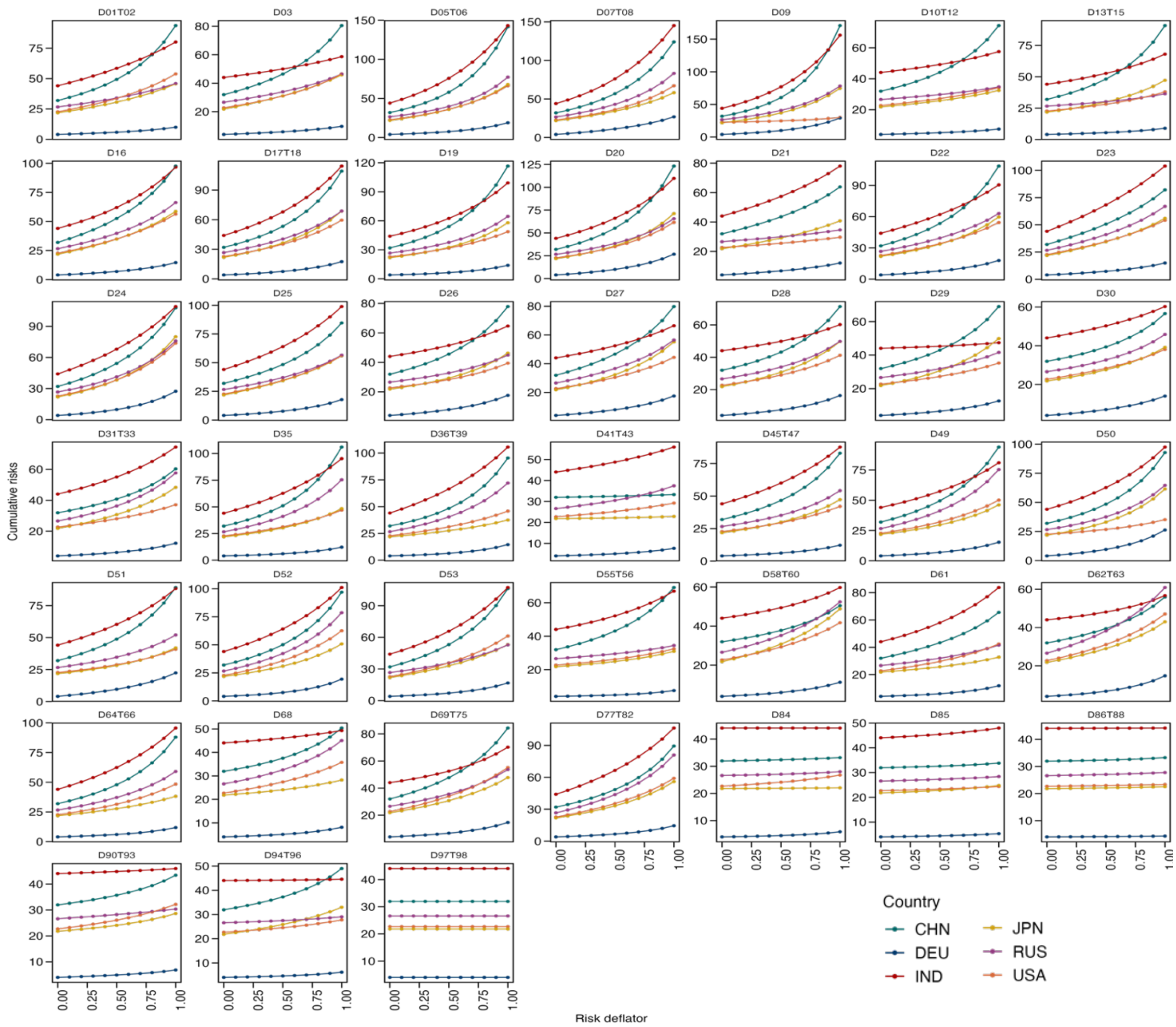


Figure 3. Risk level of each sector in major economies (γ $ragning\ from$ 0 to 1).

To compare the production chain length and the expected risks, Figure 4 shows the estimated total risks and the expected number of steps before absorption for each country-sector under complete penetration of risks along GVCs. Comparing the risks and length, we find that risks are more related to country-specific characteristic, while length are more related to sector-specific risks. Notwithstanding, longer production chains do increase the risks. For instance, the D05T06, D07T08, D09 sectors in China, India, Indonesia, Hongkong have both longer production chains and higher risks compared to other countries or sectors.

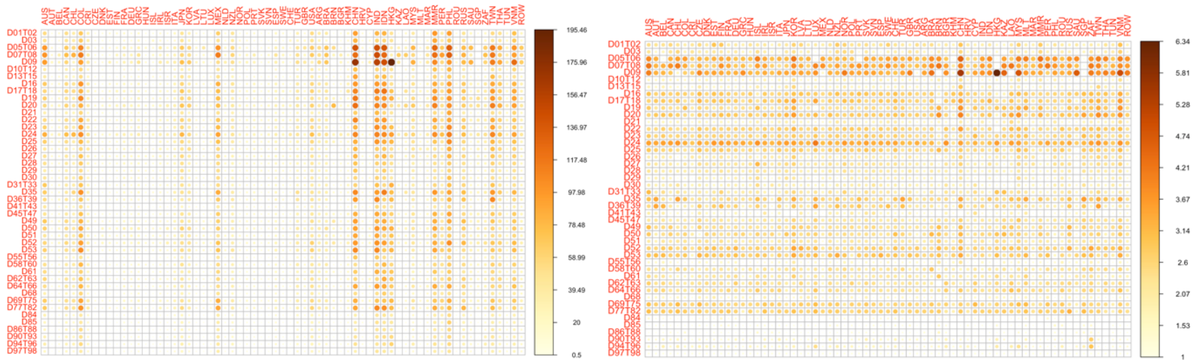


Figure 4. Country-sector expected total risks and expected number of steps before reaching absorbing state ($\gamma = 1$). a, Total expected risks estimated using WRI. b, total number of steps before absorption using Identity matrix.

3.2 Risk for each country-sector by passing-through various countries ($R_{i(k)}$)

Then we analysed how risk dynamics are transmitted through various stages of international production—how each country-sectors total expected risks are formulated by passing-through different production stages (setting $\gamma = 1$). We divided the production stages into three categories: risks inherent in the originating country-sector, risks linked to sectors within the same country as the originating sector, and risks from foreign country-sectors. Figure 5 illustrates the top 50 high-risk country-sectors, detailing their risk composition and the influence of other high-risk country-sectors. For each country-sector, the percentage contribution of foreign production stages to their total risk profile is quantified. The countries with the highest total expected risks are concentrated in developing economies, for instance, China, India, Indonesia, and Vietnam. Figure 5 also shows pronounced variability in foreign risk contribution when examining specific sector-country risks composition. Certain sector-country combinations demonstrate a significant proportion of their risk profile attributed to foreign country-sectors, such as D05T06MMR and D09VNM. Conversely, some sector-country pairs suggest a comparatively lower share of risk coming from foreign countries, indicating a more domestically centered risk profile, such as D09HKG and D09CHN. For instance, the mining support activities sector in Vietnam (D09VNM) shows a foreign risk contribution nearing 50%, indicating a heavy reliance on international stages of production or services and a heavy exposure on international risks. This contrasts with the same sector in China (D09CHN), where the foreign risk is markedly less (about 5%), suggesting a different risk structure that may stem from distinct supply chain configurations or trade policies. This suggests that some particular sectors have a high degree of interconnectedness with the global market, potentially exposing them to transnational risks. Figure 6 presents the world's top 50 risky country-sectors and show how their risks are formulated by passing-through other risky country-sectors.

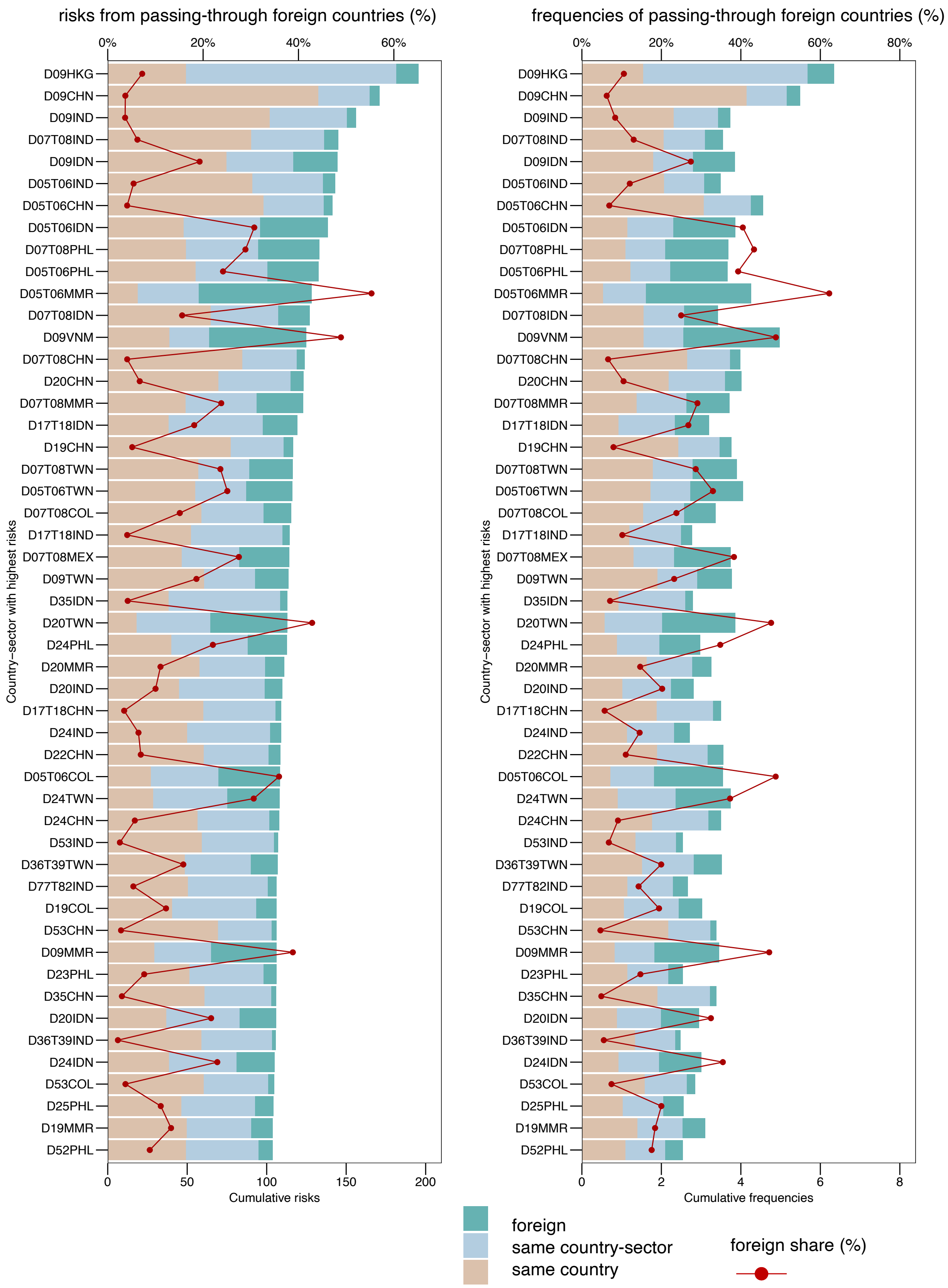


Figure 5. Top 50 risky country-sectors and their composition by passing-through other risky country-sectors ($\gamma = 1$).

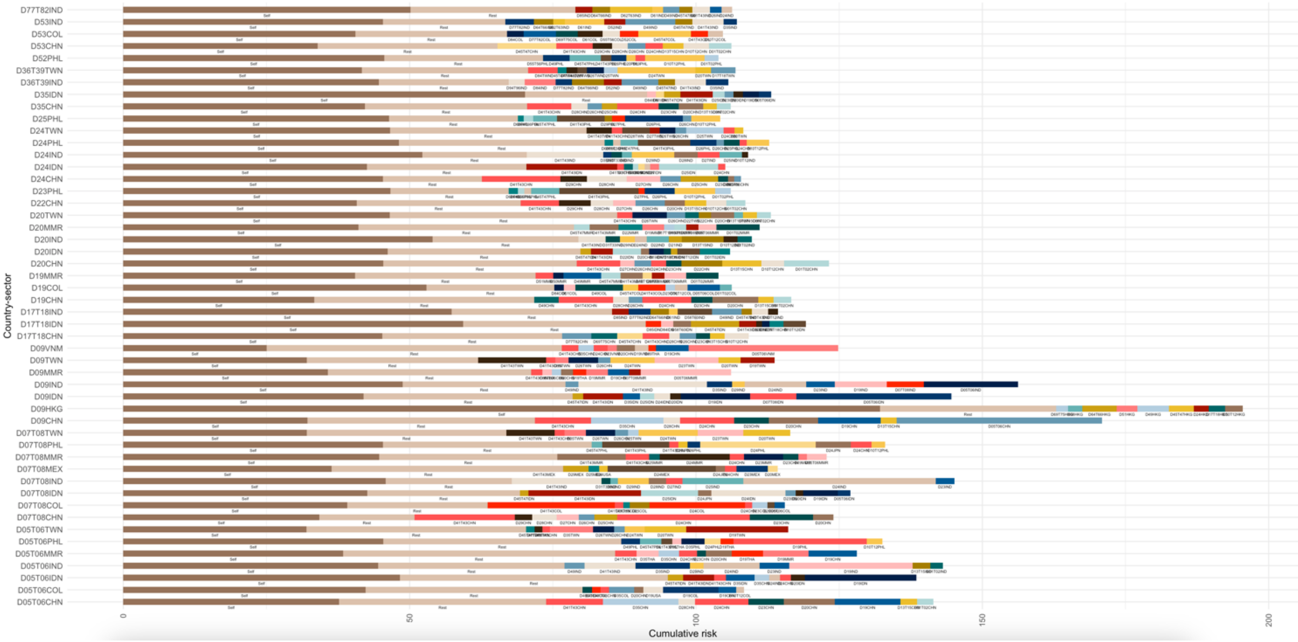


Figure 6. Top 50 risky country-sectors and their composition by passing-through other risky country-sectors ($\gamma = 1$).

3.3 Risk for each country-sector starting-absorbing pair (R_{ij})

To uncover how risks are linked to the starting point of the GVCs—the start of all the stages in each production chain, and the absorbing point of the GVCs—where the final product is finished, we estimate the risk for each country-sector starting-absorbing pairs. Figure 7 presents the top 100 risky pairs by their origin country-sector and final product country-sector. It shows that the riskiest pairs are concentrated in some countries. For instance, those GVCs originated from Australia, United States, China, Russia，and those developing countries included in ROW, and those GVCs whose final products are absorbed in India, China, and United States are related to higher risks. The GVCs started from the mining sector (D05T06) is usually associated with higher risks, for instance, the GVCs starting from D05T06 in Australia, Russia, Saudi Arabia, and ROW—which are absorbed in many sectors in China and India —already account for a large share of the top 100 GVC pairs. This is because the mining sector is related to raw material inputs, which is one of the most upstream sectors that need to pass through more stages before they finally get absorbed. Higher risks are often related to longer GVCs; however, this is not absolute. Comparing Figure 7 and Figure 8, a large part of the top risky country-sector pairs are among the top lengthy country-sectors pairs. The reason behind is that the more stages the production chains pass through, the higher the risks accumulate. However, as we can see from Figure 7 and Figure 8, some of the longest GVCs are not within the highest risks. The risk level of a GVC route is not only decided by its length, but also other factors like the sector's own characteristics, specific countries of different risk levels it passes through.

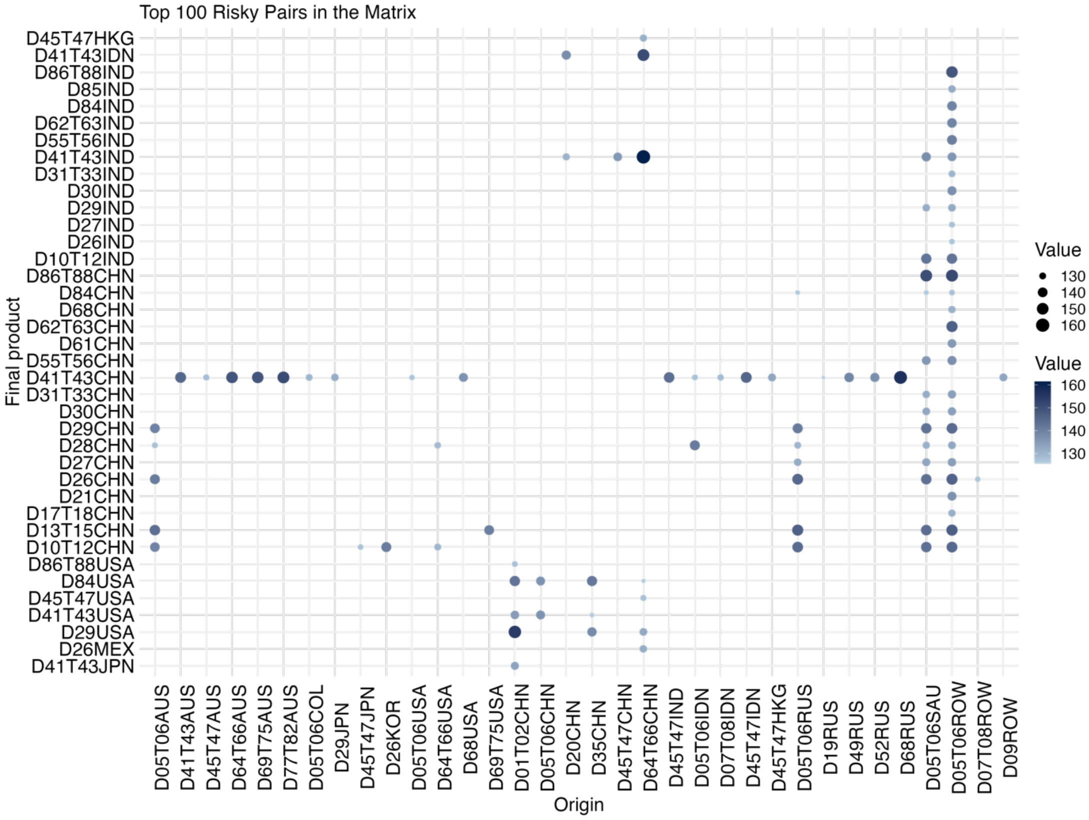


Figure 7. Top 100 risky country-sector pairs (γ = 1, value-added embodied >= 500,000\$, only GVCs started and absorbed in different countries).

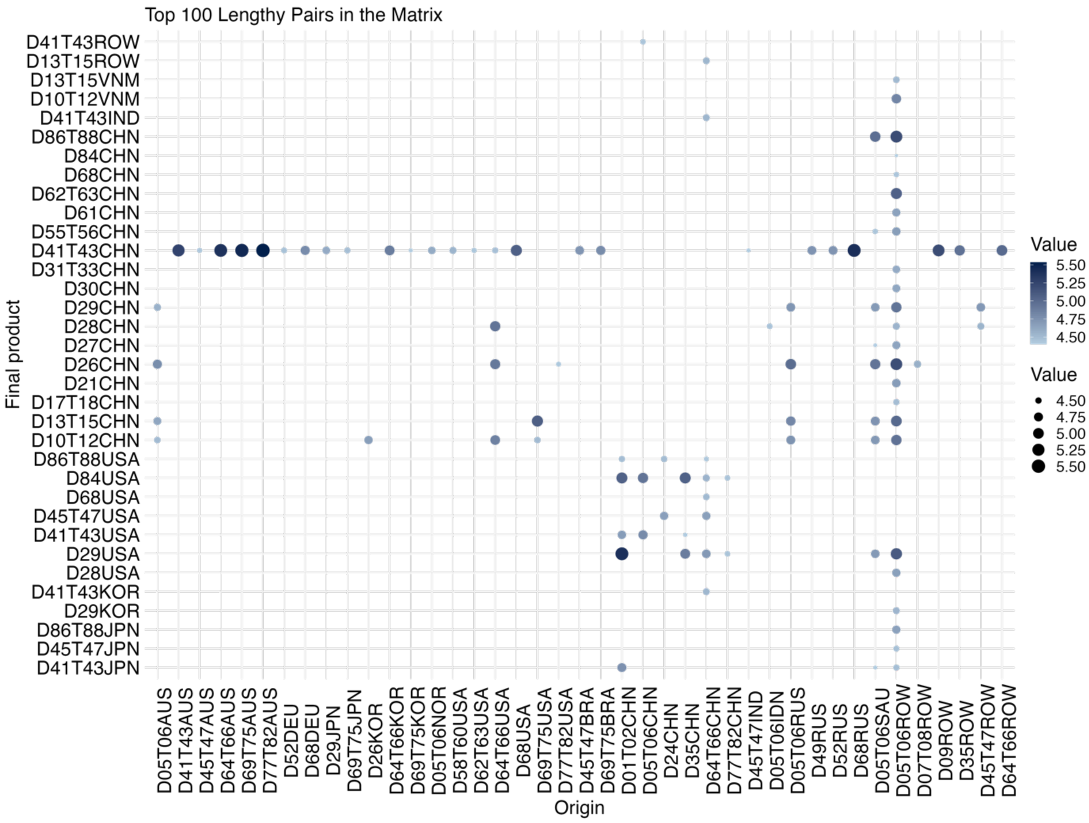


Figure 8. Top 100 lengthy country-sector pairs ($\gamma = 1$, value-added embodied >= 500,000$, only GVCs started and absorbed in different countries).

Figure 9 further shows the correlation between risk and length (distance between starting point and absorbing point) for the GVCs that are started and absorbed in different countries and above 500,000$. It shows a positive relationship between GVC risk and length. It is worth noticing that Figures 7, 8, and 9 are all based on the assumption the $\gamma = 1$, where the risk of each production stage can fully accumulate. If we suppose a smaller value for γ, the results can be different. For instance, when $\gamma = 0$, only the current stage risk matters and no accumulation effect or risk penetration will happen.

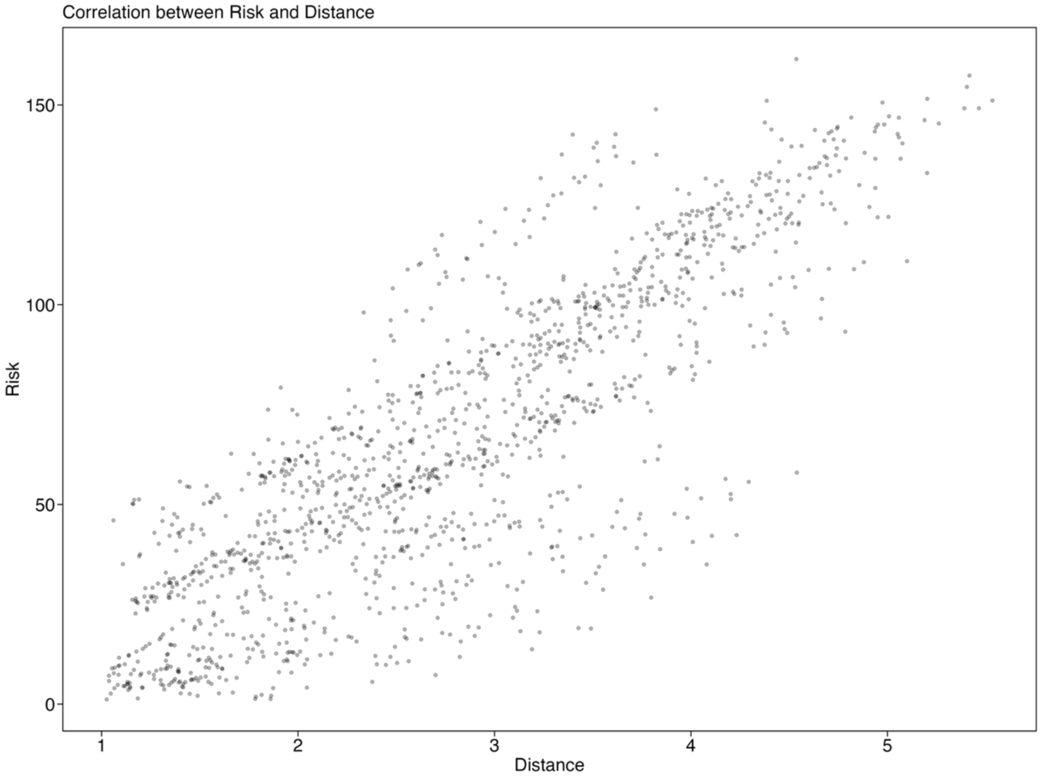
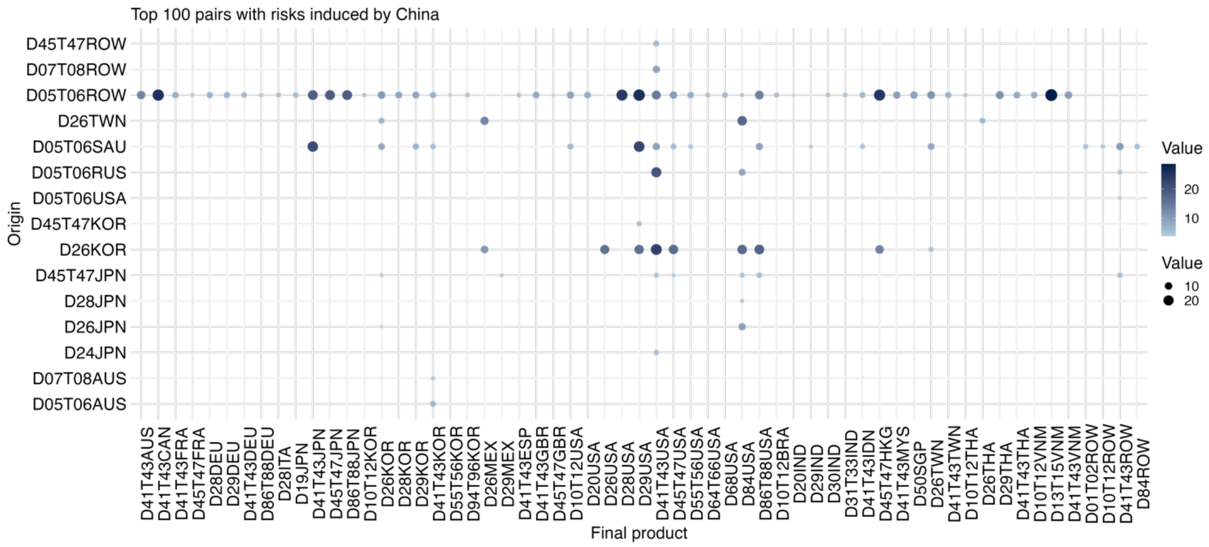


Figure 9. Bilateral GVC risk-length correlation ($\gamma = 1$, value-added embodied >= 500,000$, only GVCs started and absorbed in different countries).

3.4 Risk for each country-sector starting-absorbing pair by passing-through various countries ($R_{ij(k)}$)

This section uncovers how the risk a bilateral starting-absorbing pair depend on the specific production stages it passes through. Even if both the starting country and the absorbing country do not have direct trade with the passing-through country, the complex GVCs that link these countries together will induce extra risks. As the risks level in WRI are identical across sectors within a given country, this study analyses how passing through of different countries contribute to the GVC risks.


Top 100 pairs with risks induced by China
Origin
D45T47ROW
D07T08ROW
D05T06ROW
D26TWN
D05T06SAU
D05T06RUS
D05T06USA
D45T47KOR
D26KOR
D45T47JPN
D28JPN
D26JPN
D24JPN
D07T08AUS
D05T06AUS
Final product
D41T43AUS
D41T43CAN
D41T43FRA
D45T47FRA
D28DEU
D29DEU
D41T43DEU
D86T88DEU
D28ITA
D19JPN
D41T43JPN
D45T47JPN
D86T88JPN
D10T12KOR
D26KOR
D28KOR
D29KOR
D41T43KOR
D55T56KOR
D94T96KOR
D26MEX
D29MEX
D41T43ESP
D41T43GBR
D45T47GBR
D10T12USA
D20USA
D26USA
D28USA
D29USA
D41T43USA
D45T47USA
D55T56USA
D64T66USA
D68USA
D84USA
D86T88USA
D10T12BRA
D20IND
D29IND
D30IND
D31T33IND
D41T43IDN
D45T47HKG
D41T43MYS
D50SGP
D26TWN
D41T43TWN
D10T12THA
D26THA
D29THA
D41T43THA
D10T12VNM
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D01T02ROW
D10T12ROW
D41T43ROW
D84ROW
Value
20
10
Value
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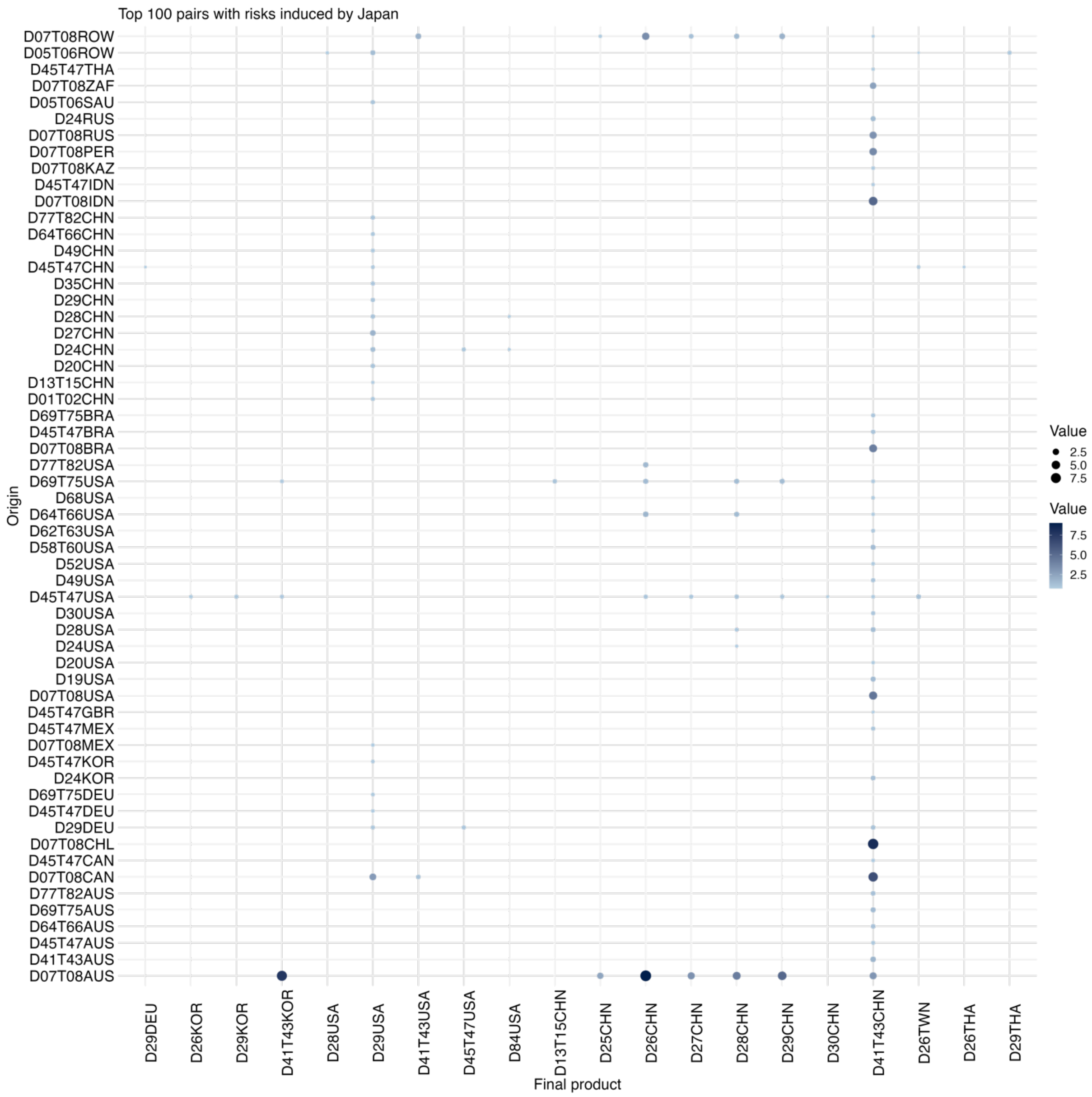
20


Top 100 pairs with risks induced by Japan
Origin
D07T08ROW
D05T06ROW
D45T47THA
D07T08ZAF
D05T06SAU
D24RUS
D07T08RUS
D07T08PER
D07T08KAZ
D45T47IDN
D07T08IDN
D77T82CHN
D64T66CHN
D49CHN
D45T47CHN
D35CHN
D29CHN
D28CHN
D27CHN
D24CHN
D20CHN
D13T15CHN
D01T02CHN
D69T75BRA
D45T47BRA
D07T08BRA
D77T82USA
D69T75USA
D68USA
D64T66USA
D62T63USA
D58T60USA
D52USA
D49USA
D45T47USA
D30USA
D28USA
D24USA
D20USA
D19USA
D07T08USA
D45T47GBR
D45T47MEX
D07T08MEX
D45T47KOR
D24KOR
D69T75DEU
D45T47DEU
D29DEU
D07T08CHL
D45T47CAN
D07T08CAN
D77T82AUS
D69T75AUS
D64T66AUS
D45T47AUS
D41T43AUS
D07T08AUS
Final product
D29DEU
D26KOR
D29KOR
D41T43KOR
D28USA
D29USA
D41T43USA
D45T47USA
D84USA
D13T15CHN
D25CHN
D26CHN
D27CHN
D28CHN
D29CHN
D30CHN
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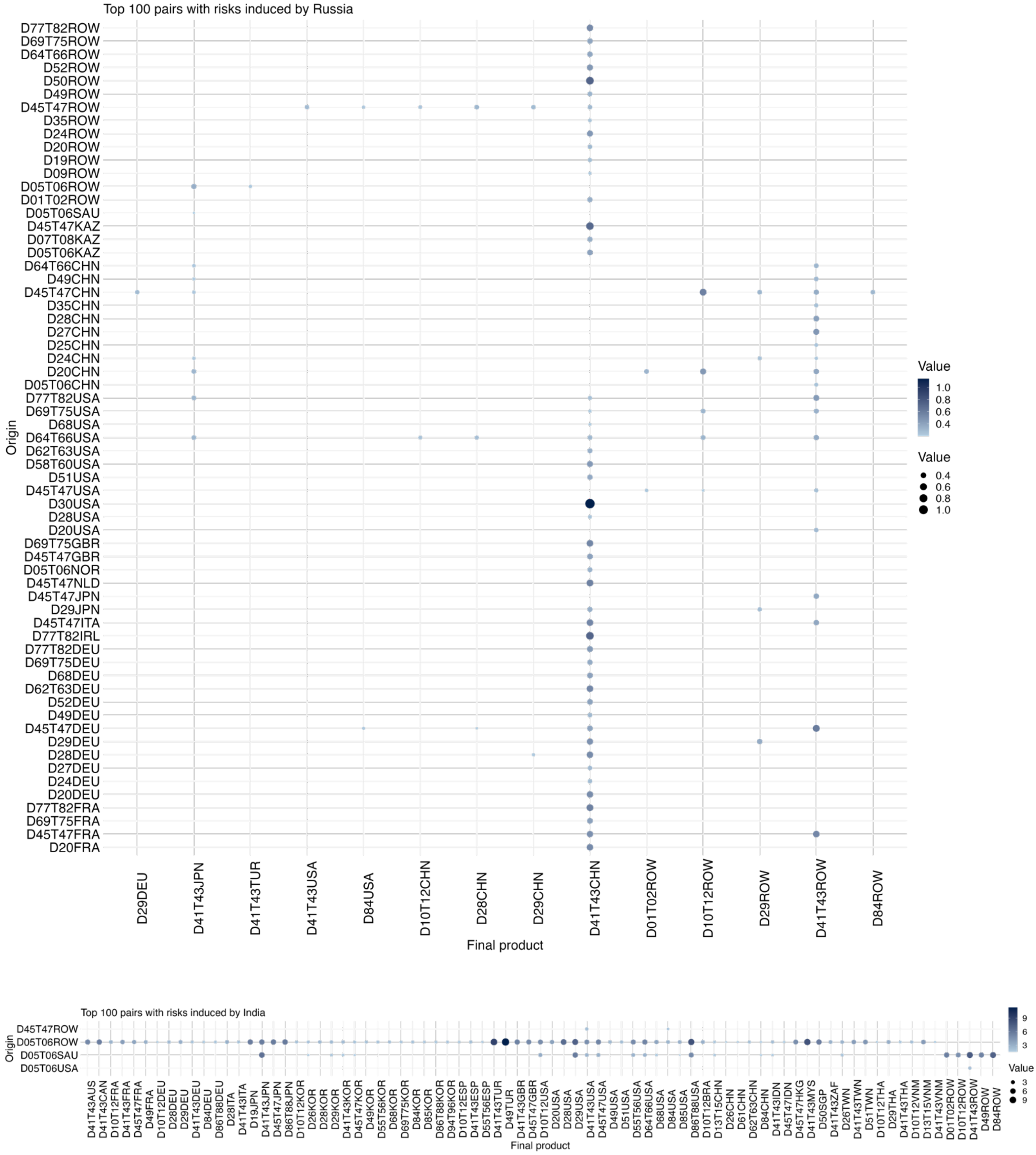

Figure 10. Country-sector pairs with the highest risk induced by passing through the country. a, China. b, Japan. c, Russia. d, India. ($\gamma = 1$, value-added embodied >= 500,000$, only GVCs started and absorbed in foreign countries).

Figure 10 shows the top 100 country-sector starting-absorbing pairs that have risks induced by China, Japan, Russia, and India. China, Japan, Russia, and India are selected for different reasons. As China is one the countries that are at the core of the GVCs, it is important to see how passing through China can contribute to GVC risk. Japan is one of the countries that are highly exposed to the shock of earthquakes, and previous studies have confirmed that earthquakes in Japan could influence the world along GVCs. Russia is analysed not only because it is one of the world's largest energy providers, but also due to its geopolitical tensions. India is one the largest developing countries, with huge amount of labor force vulnerable to natural disasters—floods, heatwaves, droughts, etc.—and inadequate facilities to cope with such issues. Figure 10 shows that the distribution of the country-sector pairs with the highest risk due to each country is largely different in the four economies. For instance, most of the top 100 pairs with highest risk due to passing-through China are started in the mining sector (D05T06) in the ROW, the mining sector in Saudi Arabia, or the ICT sector (D26) in Korea. However, these top 100 GVC pairs are related to final products in much more countries and sectors, which is more dispersed. For instance, the construction sector (D41T43) in Japan, the ICT sector (D26) in Korea, the motor vehicle sector (D29) in the United States, the wholesale and retail trade sector (D45T47) in the United States, as well as many sectors in India, Vietnam, and Thailand. This is not only because China plays the role of "world factory" that connects all these participants in GVCs, but also because GVCs starting from these country-sectors and absorbed as final product in the relevant countries passes through China more frequently. The results are similar in India, where the highest risks induced are started from the mining sector (D05T06) in the ROW and absorbed in a large variety of countries and sectors. However, the GVC pairs with the highest risk induced by passing through Russia and Japan distribute differently—they start from many different countries but end only in a few country-sectors. For instance, most of the risks induced by passing-through Russia are related to GVCs finally absorbed in the construction sector (D41T43) in China.

To analyse which countries have induced the most risks in all GVCs, this study furthers shows the sum of the risks for all GVC routes induced by passing through each country. This is equivalent to the average risk for a GVC of passing-through any country, where the latter one can be obtained by dividing the relevant values in Figure 11 by 9090225 (3015×3015)—the number of starting-absorbing GVC combinations. Figure 12 presents the frequency of all GVCs of passing-through each country, which is equivalent to a global universal unit risk index. Comparing Figure 11 and Figure 12, it shows that both the passing-through frequency and the countries' own risk level influence the total risk of passing-through this country. For those countries with high passing-through frequencies, such as China, Germany, Japan, Korea, United States, it is critical to control their influence on global total risks, either by reducing their own risk level through adaptation and capacity building, or by improving the resilience of the GVCs.

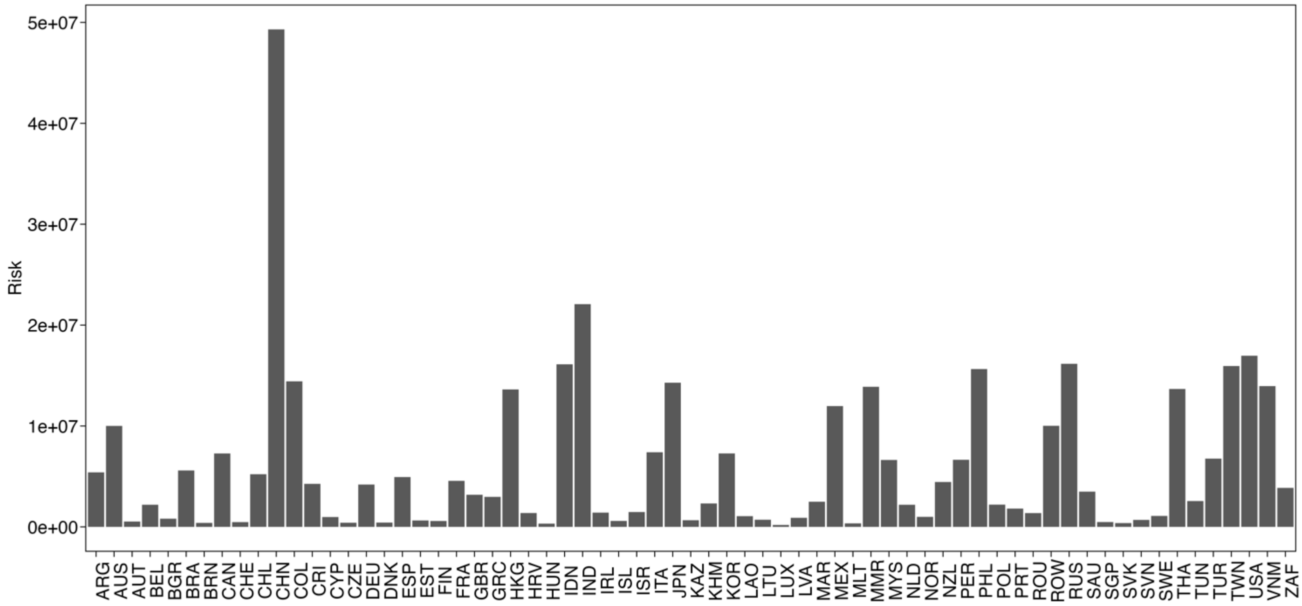

Figure 11. Total risks for all GVC routes induced by passing through each country ($\gamma = 1$).

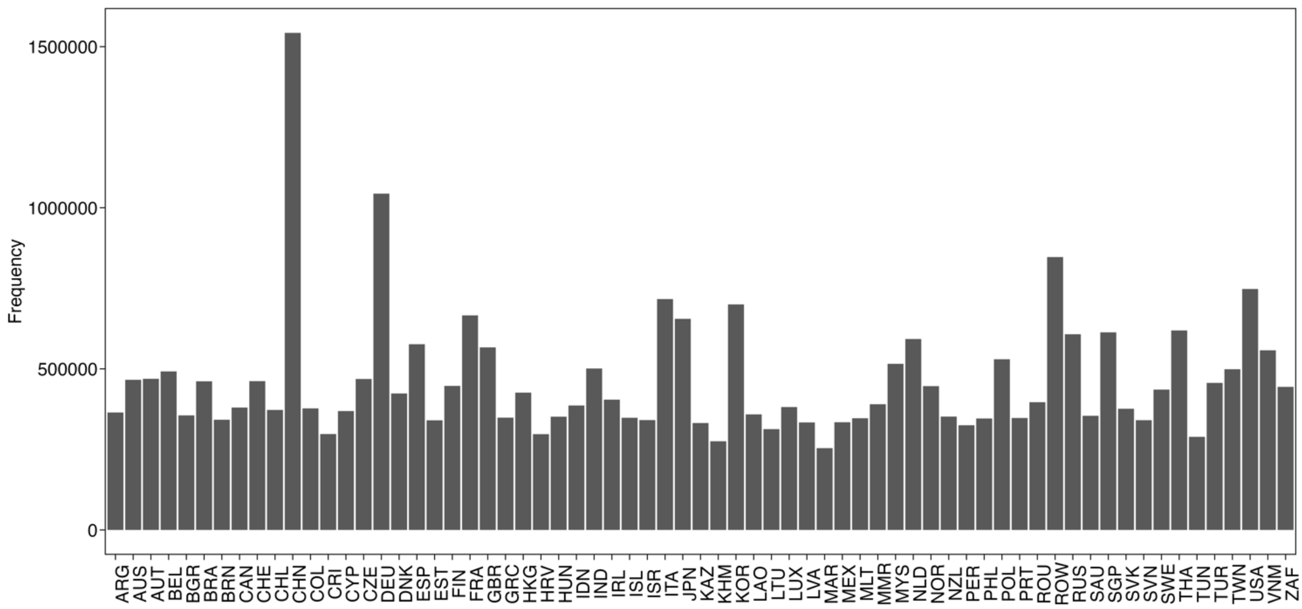

Figure 12. Total frequencies for all GVC routes passing through each country ($\gamma = 1$).

4. Discussions

This study tries to shed light on measuring GVCs risks by the development and application of an Absorbing Markov Model with Rewards. Such a model can contribute to quantifying the complexities of GVCs and understanding how the risks are accumulated and transmitted in GVCs from the probability-theory perspective. This method can trace the flow of goods and services, quantify the transmission of risks across these networks, and highlight the states/stages that brings in the highest risks to the system. This method has several advantages. First, it is consistent with previous important studies in GVCs, such as the input-output model/extended input-output model/footprint analysis, the GVC length measurement, upstreamness and downstreamness measurement, as well as the pass-through frequency measurement. This makes the results of this model more understandable and interpretable. Second, this model can be applied to measure various risk factors—natural disasters, climate change, energy shocks, geopolitical tensions, as well as many other aspects—within a unified analytical framework. This allows us to embed all kinds of risk factors in the analytical framework, thus, the risks network is dependent on both the GVC structure and the risks at each state. Third, by introducing a risk deflator index, this model is able to capture how risks are penetrating and accumulating in the GVCs. The variability of the risk deflator index brings more flexibility to the model, which is able to capture the human's ability to control the risk, reduce vulnerability, and bolster resilience. This application of this model can provide policymakers, businesses, and economists with a tool to assess vulnerabilities within GVCs and devise targeted strategies to against a spectrum of global challenges.

In an era of GVCs, the risk discount index emerges as a pivotal factor in shaping the resilience and vulnerability of international trade networks. This index, with values ranging from 0 to 1, acts as a barometer for the accumulation of risks along the supply chain. When the risk discount index is smaller, entities are impacted by the risks associated with their direct or close production stages, isolating them from the broader network of supply chain vulnerabilities. Conversely, when the risk discount index is larger, it suggests a more integrated risk model where disruptions, regardless of their position within the supply chain, can aggregate and intensify as they propagate through the network. The trend towards more specialized GVCs in recent decades brings to light the critical nature of role in risk management. As supply chains become more complex and longer, the potential for risk accumulation increases, highlighting the necessity for risk mitigation strategies. Addressing this challenge calls for careful risk management measures. For instance, one measure is to monitor the supply chain dynamically to create a more resilient supply chain. When risks happen at one state along the GVCs, all related GVC participants can be warned of such sudden risk and get prepared to reduce their exposure. Other risk mitigation strategies include real-time risk assessment, strategic stockpiling of essential components, and fostering collaborative risk management practices among supply chain participants. With the collaboration of participants in the GVCs, stakeholders can bolster the robustness of GVCs and get better equipped to face the complexities and uncertainties of global trade.

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Appendix

Appendix A. Upstreamness and downstreamness

Linking the upstreamness with the extent to which a country-industry pair sells its output for final use to consumers worldwide or instead sells intermediate inputs to other producing sectors in the world economy, the upstreamness in Antràs et al. (2012) can be expressed as follows,

$$\boldsymbol{U}^A = \frac{\boldsymbol{F} + 2\boldsymbol{AF} + 3\boldsymbol{A}^2\boldsymbol{F} + \cdots}{\boldsymbol{X}} = \frac{(\boldsymbol{I} + 2\boldsymbol{A} + 3\boldsymbol{A}^2 + \cdots)\boldsymbol{F}}{(\boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \cdots)\boldsymbol{F}} = \frac{\boldsymbol{LLF}}{\boldsymbol{LF}} = \frac{\boldsymbol{LX}}{\boldsymbol{X}}$$

The upstreamness proposed by Fally (2012) is a measure of distance from final use—based on the notion that industries selling a disproportionate share of their output to relatively upstream industries should be relatively upstream themselves.

$$\boldsymbol{U}^f = \boldsymbol{\xi} + \boldsymbol{BU}^f$$

$$\boldsymbol{U}^f = (\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{\xi}$$

Proof

In this section, we will try to prove the upstreamness measures proposed by Antràs et al. (2012), Fally (2012), and this paper are equivalent. From the AMCR model proposed in this paper, we know the average absorption time or the average number of steps from starting state before absorption can be calculated as follows,

$$\boldsymbol{\tau} = \boldsymbol{N\xi} = (\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{\xi} = \boldsymbol{U}^f$$

That is, the upstreamness in the AMCR model is exactly the upstreamness index proposed by Fally (2012). Next, we show that $\boldsymbol{U}^A$ and $\boldsymbol{U}^f$ are also equivalent.

$$\boldsymbol{U}^A = \frac{\boldsymbol{LX}}{\boldsymbol{X}} = \widehat{\boldsymbol{X}}^{-1}\boldsymbol{L}\widehat{\boldsymbol{X}}\boldsymbol{\xi}$$

Given we already known that $\boldsymbol{B} = \widehat{\boldsymbol{X}}^{-1}\boldsymbol{A}\widehat{\boldsymbol{X}}$ or $\boldsymbol{A} = \widehat{\boldsymbol{X}}\boldsymbol{B}\widehat{\boldsymbol{X}}^{-1}$, we have

$$\boldsymbol{L} = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \cdots = \boldsymbol{I} + \widehat{\boldsymbol{X}}\boldsymbol{B}\widehat{\boldsymbol{X}}^{-1} + \widehat{\boldsymbol{X}}\boldsymbol{B}^2\widehat{\boldsymbol{X}}^{-1} + \cdots = \widehat{\boldsymbol{X}}(\boldsymbol{I} + \boldsymbol{B} + \boldsymbol{B}^2 + \cdots)\widehat{\boldsymbol{X}}^{-1} = \widehat{\boldsymbol{X}}\boldsymbol{G}\widehat{\boldsymbol{X}}^{-1}$$

and

$$\boldsymbol{U}^A = \widehat{\boldsymbol{X}}^{-1}\widehat{\boldsymbol{X}}\boldsymbol{G}\widehat{\boldsymbol{X}}^{-1}\widehat{\boldsymbol{X}}\boldsymbol{\xi} = \boldsymbol{G\xi}$$

Thus, we prove that

$$\boldsymbol{U}^A = \boldsymbol{U}^f = \boldsymbol{\tau}$$

Appendix B. Average propagation length

According to classic definition by Dietzenbacher et al. (2005) and Dietzenbacher & Romero (2007), the length between initial inputs and final products bilaterally can be expressed in two forms. The cost-push form is as follows,

$$\boldsymbol{G} = (\boldsymbol{I} - \boldsymbol{B})^{-1}$$

$$\boldsymbol{H} = \boldsymbol{G}(\boldsymbol{G} - \boldsymbol{I})$$

$$\boldsymbol{V} = \{v_{ij}\} = \begin{cases} \dfrac{h_{ij}}{g_{ij} - \delta_{ij}}, & g_{ij} \neq \delta_{ij} \\ 0, & g_{ij} \neq \delta_{ij} \end{cases}$$

$$V^B = \frac{\boldsymbol{G}(\boldsymbol{G}-\boldsymbol{I})}{\boldsymbol{G}-\boldsymbol{I}} = \frac{\boldsymbol{GG}-\boldsymbol{G}}{\boldsymbol{G}-\boldsymbol{I}}$$

While the demand-pull form can be expressed as follows,

$$\boldsymbol{V}^A = \frac{\boldsymbol{L}(\boldsymbol{L}-\boldsymbol{I})}{\boldsymbol{L}-\boldsymbol{I}} = \frac{\boldsymbol{LL}-\boldsymbol{L}}{\boldsymbol{L}-\boldsymbol{I}}$$

Proof 1

When the initial state is considered in the AMCR model proposed in this paper,

$$\boldsymbol{N}_a\boldsymbol{\xi} = \widehat{\boldsymbol{d}_a}^{-1}(\boldsymbol{I}-\boldsymbol{B})^{-1}\widehat{\boldsymbol{d}_a}\boldsymbol{\xi} = \widehat{(\boldsymbol{GF})_a}^{-1}\boldsymbol{G}\widehat{(\boldsymbol{GF})_a}\boldsymbol{\xi} = \widehat{\boldsymbol{GF}_a}^{-1}\boldsymbol{G}\widehat{\boldsymbol{GF}_a}\boldsymbol{\xi} = \frac{\boldsymbol{Gd}_a}{\boldsymbol{d}_a}$$

$$\boldsymbol{N}_{full} = \frac{\boldsymbol{GD}}{\boldsymbol{D}} = \frac{\boldsymbol{G}(\boldsymbol{GP})}{\boldsymbol{GP}} = \frac{\boldsymbol{GG}}{\boldsymbol{G}} = \frac{\boldsymbol{GG}-\boldsymbol{G}}{\boldsymbol{G}} + \boldsymbol{1}$$

where $\boldsymbol{1}$ is a matrix whose elements all take the value of 1. This expression gives the same results as $\boldsymbol{V^c}+\boldsymbol{1}$ for pair ij when $i \neq j$; and the result is $\frac{h_{ii}}{g_{ii}}+1 = \frac{h_{ii}+g_{ii}}{g_{ii}}$ for pair ij when $i=j$, contrast to $\frac{h_{ii}}{g_{ii}-1}$ in $\boldsymbol{V^c}$, or $\frac{(h_{ii}+g_{ii})(g_{ii}-1)}{h_{ii}g_{ii}}$ times of the original APL.

Proof 2

When the initial state is not considered in the AMCR model proposed in this paper,

$$\begin{aligned}
\widetilde{\boldsymbol{N}}_a &= \{\boldsymbol{M}_{ia}[\tilde{n}_j]\} \\
&= \left\{\boldsymbol{M}_{ia}\left[\sum_{k=1}^{\infty}\gamma^k u_j^k r_j\right]\right\} \\
&= \left\{\sum_{k=1}^{\infty}\left(\left(1-p'^{(k)}_{ij}\right)\cdot 0 + \gamma^k p'^{(k)}_{ij} r_j\right)\right\} \\
&= \sum_{k=1}^{\infty}\left\{\gamma^k p'^{(k)}_{ij} r_j\right\} \\
&= \sum_{k=1}^{\infty}\gamma^k \boldsymbol{Q}'^k \boldsymbol{R} \\
&= \widehat{\boldsymbol{d}_a}^{-1}[(\boldsymbol{I}-\gamma\boldsymbol{Q})^{-1}-\boldsymbol{I}]\widehat{\boldsymbol{d}_a}\boldsymbol{R} \\
&= \widehat{\boldsymbol{d}_a}^{-1}[(\boldsymbol{I}-\boldsymbol{Q})^{-1}-\boldsymbol{I}]\widehat{\boldsymbol{d}_a}
\end{aligned}$$

$$\widetilde{\boldsymbol{N}}_a\boldsymbol{\xi} = \widehat{\boldsymbol{d}_a}^{-1}[(\boldsymbol{I}-\boldsymbol{B})^{-1}-\boldsymbol{I}]\widehat{\boldsymbol{d}_a}\boldsymbol{\xi} = \frac{(\boldsymbol{G}-\boldsymbol{I})\boldsymbol{d}_a}{\boldsymbol{d}_a}$$

$$\widetilde{\boldsymbol{N}}_{full} = \frac{(\boldsymbol{G}-\boldsymbol{I})\boldsymbol{GP}}{\boldsymbol{GP}} = \frac{(\boldsymbol{G}-\boldsymbol{I})\boldsymbol{G}}{\boldsymbol{G}} = \frac{\boldsymbol{GG}-\boldsymbol{G}}{\boldsymbol{G}}$$

where $\boldsymbol{P}$ is a diagonal matrix for distribution of final products.

This expression gives the same results as $\boldsymbol{V}^B$ for pair ij when $i \neq j$; and the result is $\frac{h_{ii}}{g_{ii}}$ for pair ij when $i = j$, contrast to $\frac{h_{ii}}{g_{ii}-1}$ in $\boldsymbol{V}^B$, or $\frac{g_{ii}-1}{g_{ii}}$ times of the original APL.

Proof 3

When the initial state is not considered, we get the APL defined by Dietzenbacher et al. (2005),

$$\boldsymbol{APL}_{ij} = \frac{1\boldsymbol{A} + 2\boldsymbol{A}^2 + 3\boldsymbol{A}^3 + \cdots}{\boldsymbol{A} + \boldsymbol{A}^2 + \boldsymbol{A}^3 + \cdots} = \frac{(\boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \cdots)(\boldsymbol{A} + \boldsymbol{A}^2 + \boldsymbol{A}^3 + \cdots)}{\boldsymbol{L} - \boldsymbol{I}} = \frac{\boldsymbol{L}(\boldsymbol{L} - \boldsymbol{I})}{\boldsymbol{L} - \boldsymbol{I}}$$

When the initial state is considered, we get

$$\boldsymbol{APL'}_{ij} = \frac{\boldsymbol{I} + 2\boldsymbol{A} + 3\boldsymbol{A}^2 + 4\boldsymbol{A}^3 + \cdots}{\boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \boldsymbol{A}^3 + \cdots} = \frac{(\boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \cdots)(\boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \cdots)}{\boldsymbol{L}} = \frac{\boldsymbol{L}\boldsymbol{L}}{\boldsymbol{L}}$$

Considering this from the cost-push perspective, we get

$$\boldsymbol{APL''}_{ij} = \frac{\boldsymbol{I} + 2\boldsymbol{B} + 3\boldsymbol{B}^2 + 4\boldsymbol{B}^3 + \cdots}{\boldsymbol{I} + \boldsymbol{B} + \boldsymbol{B}^2 + \boldsymbol{B}^3 + \cdots} = \frac{(\boldsymbol{I} + \boldsymbol{B} + \boldsymbol{B}^2 + \cdots)(\boldsymbol{I} + \boldsymbol{B} + \boldsymbol{B}^2 + \cdots)}{\boldsymbol{B}} = \frac{\boldsymbol{G}\boldsymbol{G}}{\boldsymbol{G}}$$

This is exactly the same result from the AMCR method.

Appendix C. Passing-through frequency

According to (Inomata & Hanaka, 2021), the risk of global supply chains can be estimated by how many times the supply chains have pass-through specific country-sectors. The pass-through frequency can be calculated as the total impacts delivered from i to j through t as follows

$$\boldsymbol{f}_{ij(t)}^{A} = \frac{\boldsymbol{L}\boldsymbol{J}_t\boldsymbol{L} - \boldsymbol{J}}{\boldsymbol{L} - \boldsymbol{I}}, \boldsymbol{f}_{ij(t)}^{B} = \frac{\boldsymbol{G}\boldsymbol{J}_t\boldsymbol{G} - \boldsymbol{J}_t}{\boldsymbol{G} - \boldsymbol{I}}$$

where $\boldsymbol{J} = \boldsymbol{J}_t$ is an $mn \times mn$ matrix containing 1 for the (t, t)-th element and 0 elsewhere.

Proof

When the initial state is considered in the AMCR model proposed in this paper, let $\gamma = 1$ and $\boldsymbol{R} = \boldsymbol{J}_t$, the risk from initial i to j through t can be expressed as follows,

$$\boldsymbol{N}_{ij(t)}\boldsymbol{\xi} = \widehat{\boldsymbol{d}_j}^{-1}(\boldsymbol{I} - \boldsymbol{B})^{-1}\widehat{\boldsymbol{d}_j}\boldsymbol{J}_t\boldsymbol{\xi} = \frac{\boldsymbol{G}\widehat{\boldsymbol{d}_j}\boldsymbol{J}_t\boldsymbol{\xi}}{\boldsymbol{d}_j} = \frac{\boldsymbol{G}\boldsymbol{J}_t\widehat{\boldsymbol{d}_j}\boldsymbol{\xi}}{\boldsymbol{d}_j} = \frac{\boldsymbol{G}\boldsymbol{J}_t\boldsymbol{d}_j}{\boldsymbol{d}_j}$$

$$\boldsymbol{N}_{ij(t)full} = \frac{\boldsymbol{G}\boldsymbol{J}_t\boldsymbol{D}}{\boldsymbol{D}} = \frac{\boldsymbol{G}\boldsymbol{J}_t(\boldsymbol{G}\boldsymbol{P})}{\boldsymbol{G}\boldsymbol{P}} = \frac{\boldsymbol{G}\boldsymbol{J}_t\boldsymbol{G}}{\boldsymbol{G}}$$

where $\boldsymbol{P}$ is a diagonal matrix.

When $i = j = t$, the expression gives $\frac{k_{ii}}{g_{ii}}$ while $\boldsymbol{f}_{ij(t)}^{A}$ equals $\frac{k_{ii}-1}{g_{ii}-1}$. Otherwise ($i \neq j, and\ i \neq t\ or\ j \neq t$), this expression gives the same results as $\boldsymbol{f}_{ij(t)}^{A}$ for pair ij passing-through t.

According to (Inomata & Hanaka, 2021), the terms $\boldsymbol{J}_t$ and $\boldsymbol{I}$ are respectively subtracted from the numerator and the denominator in $\boldsymbol{f}_{ij(t)}^{A}$ in order to negate the values corresponding to the initial final demands, which are analytically irrelevant to identifying the structure of the networks

Appendix D. Industry Classification

Code	**Industry**	**ISIC Rev.4**
D01T02	Agriculture, hunting, forestry	01, 02
D03	Fishing and aquaculture	03
D05T06	Mining and quarrying, energy producing products	05, 06
D07T08	Mining and quarrying, non-energy producing products	07, 08
D09	Mining support service activities	09
D10T12	Food products, beverages and tobacco	10, 11, 12
D13T15	Textiles, textile products, leather and footwear	13, 14, 15
D16	Wood and products of wood and cork	16
D17T18	Paper products and printing	17, 18
D19	Coke and refined petroleum products	19
D20	Chemical and chemical products	20
D21	Pharmaceuticals, medicinal chemical and botanical products	21
D22	Rubber and plastics products	22
D23	Other non-metallic mineral products	23
D24	Basic metals	24
D25	Fabricated metal products	25
D26	Computer, electronic and optical equipment	26
D27	Electrical equipment	27
D28	Machinery and equipment, nec	28
D29	Motor vehicles, trailers and semi-trailers	29
D30	Other transport equipment	30
D31T33	Manufacturing nec; repair and installation of machinery and equipment	31, 32, 33
D35	Electricity, gas, steam and air conditioning supply	35
D36T39	Water supply; sewerage, waste management and remediation activities	36, 37, 38, 39

D41T43	Construction	41, 42, 43
D45T47	Wholesale and retail trade; repair of motor vehicles	45, 46, 47
D49	Land transport and transport via pipelines	49
D50	Water transport	50
D51	Air transport	51
D52	Warehousing and support activities for transportation	52
D53	Postal and courier activities	53
D55T56	Accommodation and food service activities	55, 56
D58T60	Publishing, audiovisual and broadcasting activities	58, 59, 60
D61	Telecommunications	61
D62T63	IT and other information services	62, 63
D64T66	Financial and insurance activities	64, 65, 66
D68	Real estate activities	68
D69T75	Professional, scientific and technical activities	69 to 75
D77T82	Administrative and support services	77 to 82
D84	Public administration and defence; compulsory social security	84
D85	Education	85
D86T88	Human health and social work activities	86, 87, 88
D90T93	Arts, entertainment and recreation	90, 91, 92, 93
D94T96	Other service activities	94,95, 96
D97T98	Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use	97, 98